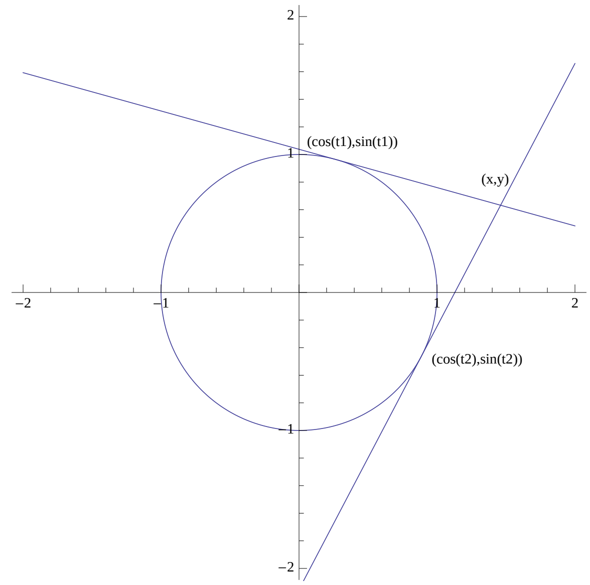
Math 2250 Homework #2

This homework assignment covers three problems (two extra credit) in the target tracking exercise.

1. PROBLEMS

1. Suppose that our detector moves on the circle $(x(t), y(t)) = (\cos t, \sin t)$. At t values t_1 and t_2 the detector registers IR light, indicating that the target is on the tangent line to the circle at that t value. Solve for the position (x, y) of the target as a function of t_1 and t_2.



Note: The answer we arrived at was

$$x = -\frac{-\sin(\mathrm{t1}) - \cos(\mathrm{t1})\cot(\mathrm{t1}) + \sin(\mathrm{t2}) + \cos(\mathrm{t2})\cot(\mathrm{t2})}{\cot(\mathrm{t1}) - \cot(\mathrm{t2})},$$

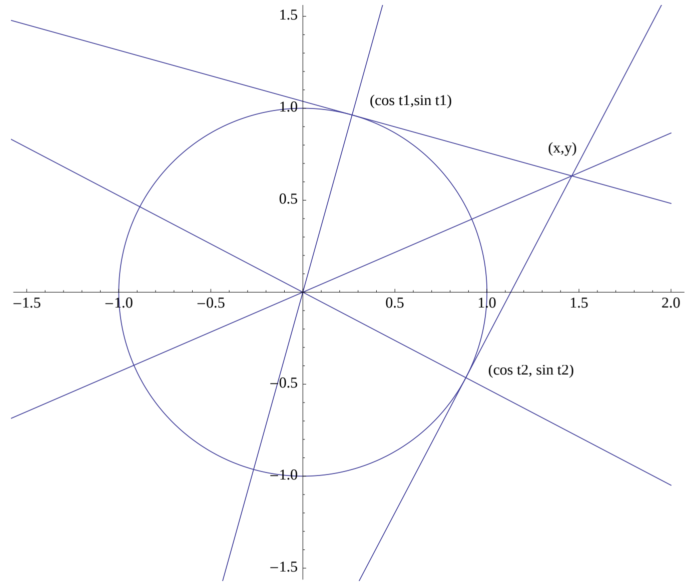
$$y = -\frac{\cos(\mathrm{t1})\cot(\mathrm{t1})\cot(\mathrm{t2}) - \cot(\mathrm{t1})\cos(\mathrm{t2})\cot(\mathrm{t2}) + \sin(\mathrm{t1})\cot(\mathrm{t2}) - \cot(\mathrm{t1})\sin(\mathrm{t2})}{\cot(\mathrm{t1}) - \cot(\mathrm{t2})}$$

2. *Mathematica* (our computer software) claims that this complicated formula actually simplifies to

$$x = \cos\left(\frac{\mathrm{t1}+\mathrm{t2}}{2}\right)\sec\left(\frac{\mathrm{t1}-\mathrm{t2}}{2}\right), \qquad y = \sin\left(\frac{\mathrm{t1}+\mathrm{t2}}{2}\right)\sec\left(\frac{\mathrm{t1}-\mathrm{t2}}{2}\right)$$

Extra credit problem, due Monday 2/7: Prove it!

(Problem 2 continued) It might help to look at the picture:



3. (Big Extra Credit Problem) Suppose that the detectors pick up a moving target. How can we solve for the trajectory of the target as a function of the interception times? For instance, suppose that the target moves along the line

$$(x(t), y(t)) = (at + b, ct + d).$$

and lies on the tangent lines to the circle at t_1, t_2, t_3, t_4. Presumably, there exists some way to solve for the unknown numbers a, b, c, d as functions (complicated trig functions) of t_1, t_2, t_3 and t_4. Give a solution!

Example: For example, for the path $(x(t), y(t)) = (12 - 0.5t, 4)$, the detectors pick up a signal at **times** $4.91659, 5.30929, 11.294, 11.8595, 17.7814, 18.6925, 24.6929, 25.8553$. Now keep in mind that one detector is mounted π radians away from the other. So these correspond to t (that is, **angle**) values $4.91659, 5.30929 - \pi, 11.294, 11.8595 - \pi, 17.7814, 18.6925 - \pi, 24.6929, 25.8553 - \pi$.

Note: I don't know the answer to this problem. I imagine that we set up and solve a system of simultaneous equations for a, b, c, and d using the t values and the formula for the tangent line to a path $(x(t), y(t))$ at $t = a$ which is

$$y - y(a) = \frac{y'(a)}{x'(a)}(x - x(a)).$$

applied to the circle $(x(t), y(t)) = (\cos t, \sin t)$.