MATH 2250

Midterm Exam II

June 20, 2014

NAME (please print legibly): _____

Your University ID Number: _____

Please complete all questions in the space provided. You may use the backs of the pages for extra space, or ask me for more paper if needed. This exam will be graded on:

- Correctness of computations.
- Clarity of explanation of procedure.
- Correctness of procedure.

A correct answer obtained using an incorrect or poorly explained procedure will not be graded for full credit. Please feel free to write as much as you like. Work carefully, and try to complete the problems you find easier before going back to the harder ones. Good luck!

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
TOTAL	70	

1. (10 points) Find the derivative of

$$f(x) = \arctan x \arcsin x.$$

ANSWER: _____

2. (10 points) The variables x and y are related by the equation

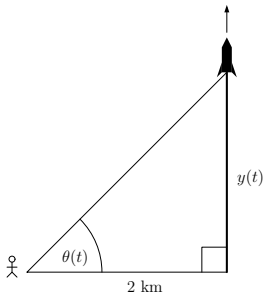
$$\sin y + 5xy = 4 \tan x$$

Find $\frac{dy}{dx}$ using implicit differentiation.

ANSWER: _____

3. (10 points)

An observer watches a rocket launch from a distance of 2 km using a surveyor's transit which enables them to measure the angle $\theta(t)$ of the line of sight between the observer and rocket as the rocket climbs.



When the angle $\theta(t)$ is $\pi/4$ radians, the angle $\theta(t)$ is increasing at a rate of 0.1 radians/second. How fast is the rocket climbing at this point? (Include units.)

(More space to continue work on the rocket problem if you need it.)

ANSWER: _____

Bonus. (5 pts) As the rocket continues to climb at the same speed, will the rate of change of the angle $\theta(t)$ increase or decrease? Why? (Notice that I'm assigning bonus points for your **explanation**, not your answer.)

ANSWER: _____

4. (10 points) A tree has a circumference of 10 inches. One year later, the circumference of the tree has grown to 12 inches. Estimate the change in the cross-sectional area of the tree using the linear approximation of the area function

$$A(c) = \frac{c^2}{4\pi}$$

at $c = 10$.

ANSWER: _____

Now compute the actual cross-sectional area $A(c)$ when $c = 12$.

ANSWER: _____

5. (10 points) Pierre-Francois Verhulst proposed in 1838 the model

$$P(N) = rN\left(1 - \frac{N}{K}\right)$$

for the rate of production P of fish in a fishery (per year) as a function of the number of fish N, the total "carrying capacity" of the fishery K, and the "intrinsic growth rate" of the fish species r.

Assume that $K = 1$ million fish and r is a positive constant.

Find the maximum of the function $P(N)$ for values of N between 1000 and $500{,}000$. This is often called the *maximum sustainable yield* of the fishery because harvesting this many fish each year will keep the population of fish constant.

ANSWER: _____

6. (10 points) The function

$$f(x) = 4x - 2x^2 - x^3 + \frac{1}{2}x^4 + \frac{1}{5}x^5$$

has derivative

$$f'(x) = (x-1)^2(x+2)^2.$$

Find the critical points of the function and classify them as local mins, local maxes, or neither.

ANSWER: _____

7. (10 points) Find a general formula for the derivative of the product of *three* functions

$$\frac{d}{dx} f(x)\, g(x)\, h(x)$$

in terms of $f(x)$, $g(x)$, $h(x)$ and their derivatives $f'(x)$, $g'(x)$ and $h'(x)$.

ANSWER: _____

Bonus (5 points): Now generalize this to give a formula for the derivative of the product of n functions $f_1(x) \ldots f_n(x)$.

ANSWER: _____