Math 2250 Homework #3

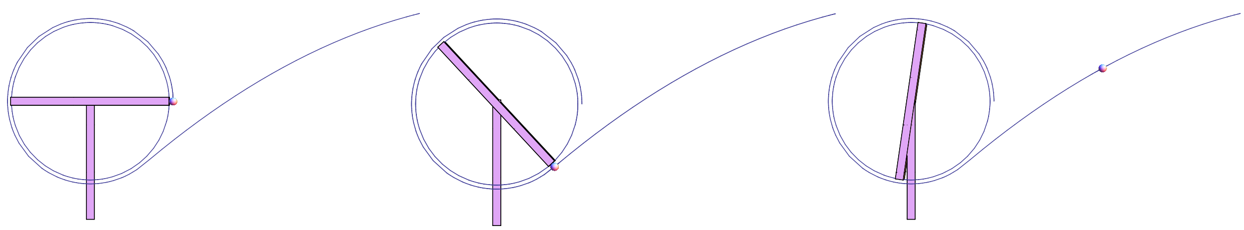
This homework assignment covers the robot range lab. The setup for the lab is that we have built a robot to throw a ball by attaching the ball to a rotating arm using an electromagnet. The arm has length 1 m (for now) and rotates around a center at $(0,0)$ at an (angular) speed of 1 radian per second (for now). The position of the arm is then given by

$$x_{\text{arm}}(\theta) = \cos\theta. \qquad y_{\text{arm}}(\theta) = \sin\theta.$$

At some angle θ_0, we release the ball. We assume that this is time $t = 0$ and that the ball then follows a parabolic path given by some functions

$$x_{\text{ball}}(t) = at^2 + bt + c. \qquad y_{\text{ball}}(t) = pt^2 + qt + r.$$

This is shown in the pictures



ball starts at $\theta = 0 \ldots$ $\qquad$ $\ldots$ released at $\theta = \theta_0 \ldots$ $\qquad$ $\ldots$ and flies along parabola.

1. Problems

1. Find $x_{\text{ball}}(t)$ and $y_{\text{ball}}(t)$ for a given θ_0. Your answer should involve various trig functions of θ_0. Keep in mind that the position and velocity of the **arm** at $\theta = \theta_0$ are the same as the position and velocity of the **ball** at time $t = 0$. Use units of meters and seconds so that the acceleration of gravity is $-9.8m/s^2$.

   Note: The answer we arrived at in class was

   $$x_{\text{ball}}(t) = -\sin\theta_0 t + \cos\theta_0. \qquad y_{\text{ball}}(t) = -4.9t^2 + \cos\theta_0 t + \sin\theta_0.$$

2. For a fixed θ_0, find the maximum value of $y_{\text{ball}}(t)$. The answer is a new function $H(\theta_0)$ which tells us how **high** the robot will throw the ball if we release at angle θ_0. This is a max/min problem where we take the derivative with respect to t.

3. Find the maximum value of $H(\theta_0)$ as θ_0 varies from 0 to 2π. This tells us the release angle which allows the robot to throw the ball *highest*.

4. Suppose the ground is located $y = -1.25$ (that is, we locate the motor on a stand so that it is 1.25 meters above the ground). For a fixed θ_0 use your $y_{\text{ball}}(t)$ function to figure out *when* the ball hits the ground. Use that time and your $x_{\text{ball}}(t)$ function to figure out *where* the ball hits the ground. The answer is a new function $R(\theta_0)$ which tells us how **far** the robot will throw the ball if we release at angle θ_0.

5. Find the maximum value of $R(\theta_0)$ as θ_0 varies from 0 to 2π. This tells us the release angle which allows the robot to throw the ball *farthest*.
6. Suppose we want to hit a target at position $(x_{\text{target}}, y_{\text{target}})$. Use your $x_{\text{ball}}(t)$ and $y_{\text{ball}}(t)$ functions to solve for the release angle (or angles!) which will cause the ball to pass through $(x_{\text{target}}, y_{\text{target}})$. You won't be able to solve the equations that you set up for any $(x_{\text{target}}, y_{\text{target}})$: what does this mean?