Quilt Patterns and Mathematics

Letting a computer inspire new quilt patterns

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Cotton Patch Quilters

Outline

1. Geometry and Quilting
   - A tour of some traditional quilts
   - Tilings
2. Modern mathematical designs
   - L-systems
   - Modern art and graphic design

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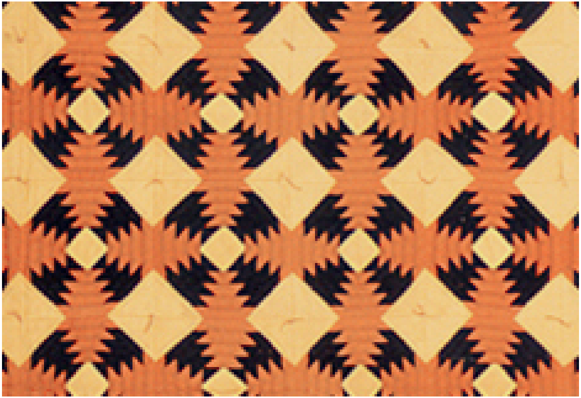
1879 quilt with grid symmetry.



www.centerforthequilt.org

1875 quilt with hexagonal symmetry.

www.centerforthequilt.org

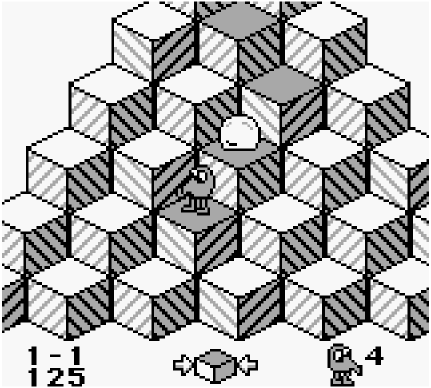
- This quilt has the same symmetry but it looks very different.

1890's box quilt.

www.centerforthequilt.org

- A box quilt has 6-way (hexagonal) symmetry.

The box quilt pattern.



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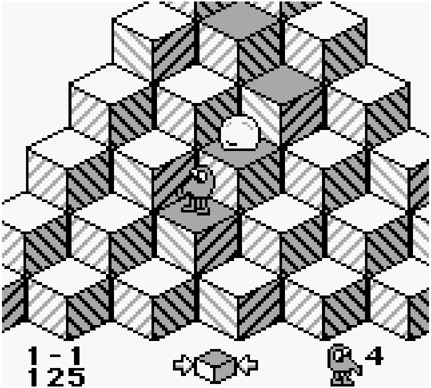
The box quilt pattern.



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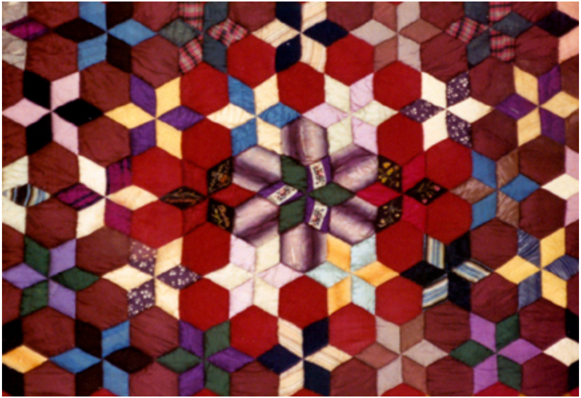
1870's quilt with hexagonal symmetry.



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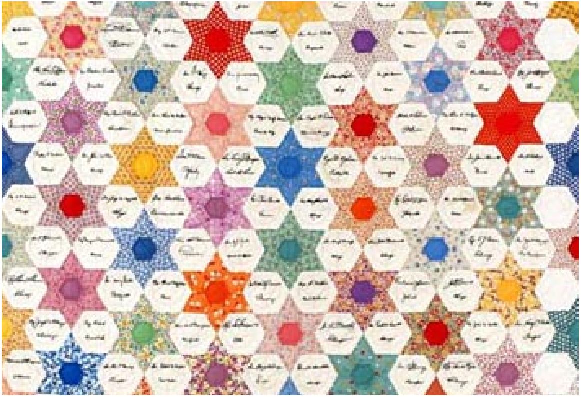
- This quilt has the same symmetry, but alternates hexagons and rhombi.

1940's quilt with the same pattern



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Many quilts are regular tessellations

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A *regular tessellation* uses a single polygon as a block, arranging the blocks with maximum symmetry.

There are only *three* regular tessellations of the plane.

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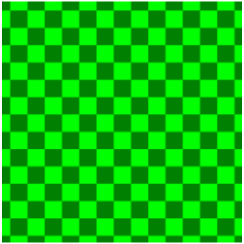

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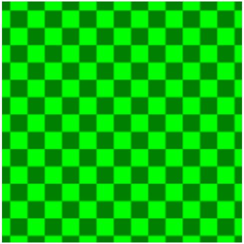

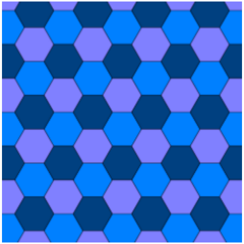

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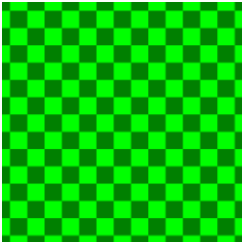
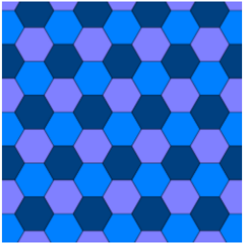
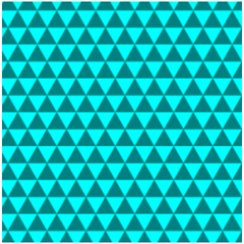






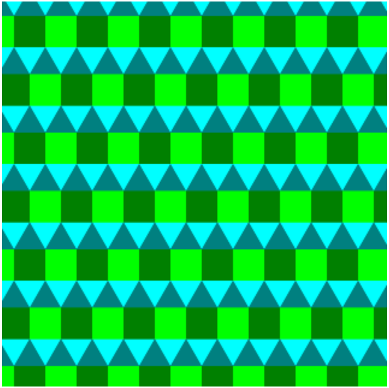
Less common are Archimedean tessellations

Definition

An *Archimedean tessellation* uses more than one polygon as a block, and requires slightly less symmetry.

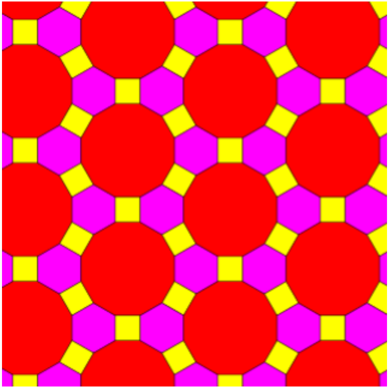
There are *eight* Archimedean tessellations of the plane.

Archimedean tessellations of the plane.



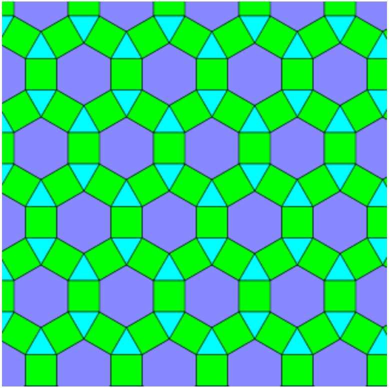
Elongated triangular tiling.

Archimedean tessellations of the plane.



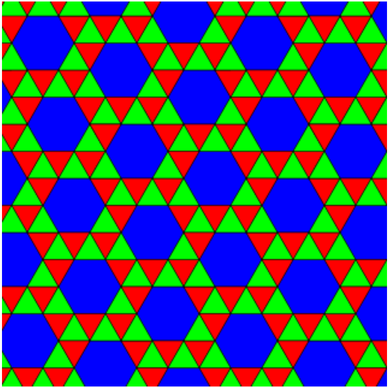
Great rhombitrihexagonal tiling.

Archimedean tessellations of the plane.



Small rhombitrihexagonal tiling.

Archimedean tessellations of the plane.



Snub hexagonal tiling.

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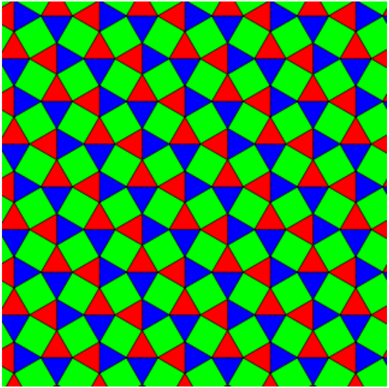
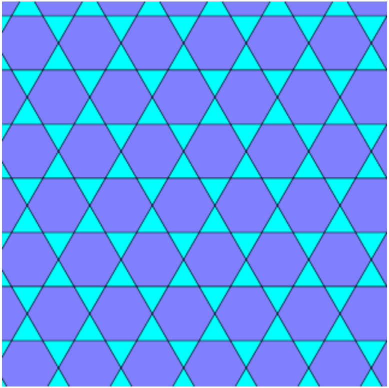


Snub square tiling.

Archimedean tessellations of the plane.



Trihexagonal tiling.

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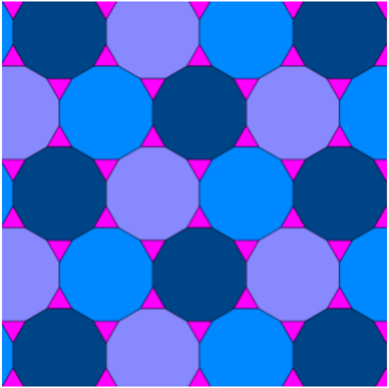

Truncated hexagonal tiling.

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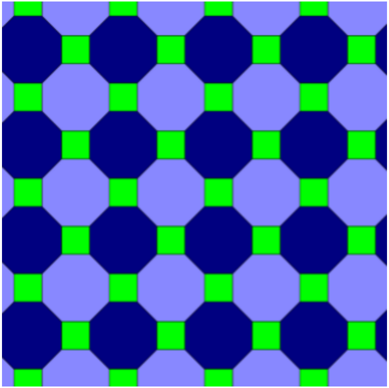

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Even more interesting tilings

Another kind of interesting tiling is the *radial* tiling. These use only one block, but arrange the block in a spiral pattern. You can do a lot of different things with only one block.

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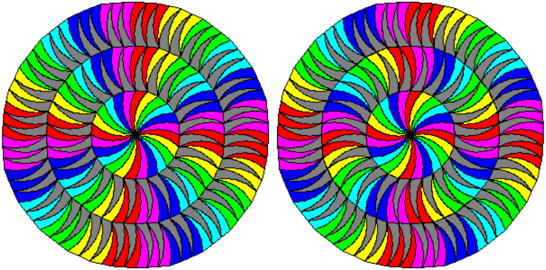

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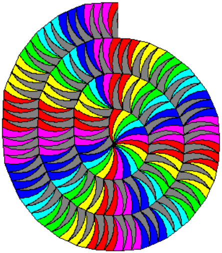

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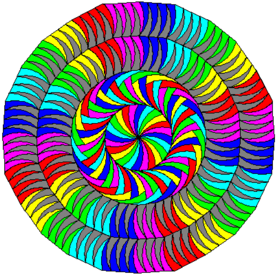

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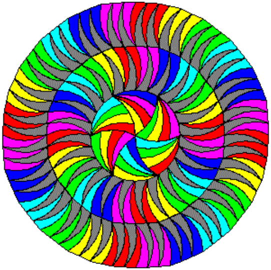
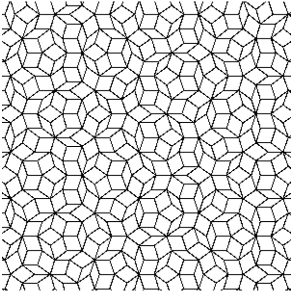


Penrose tilings

Some very interesting tilings have been discovered in the past twenty years. This one, the Penrose tiling, doesn't repeat at all. It only uses two kinds of blocks.



But really explaining this would take too long . . .

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A different kind of quilt math.

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- Understanding irregular objects requires a new kind of math.
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What is an *L*-system?

Definition

An *L*-system is a starting string of letters (the *axiom*) together with a collection of rules for replacing letters by other letters (the *derivations*). (Yikes!)

We build up a pattern from an *L*-system by repeatedly applying the rules to the initial string of letters. Suppose we start with the string a and apply the rules

$$b \rightarrow a \quad \text{and} \quad a \rightarrow ab.$$

We get a, ab, aba, $abaab$, $abaababa$, $\ldots$ and so forth.

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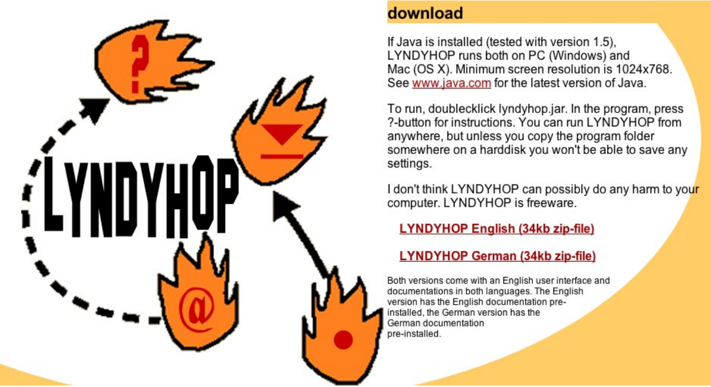
How do I make a quilt out of *that*?

We can draw pictures with L-systems by interpreting the letters as commands for drawing a pattern:

$$F \to \text{go forward one step}$$
$$+ \to \text{turn left by } \delta \text{ degrees}$$
$$- \to \text{turn right by } \delta \text{ degrees}$$

We can then interpret the series of strings as a series of designs to inspire quilting!

Let’s see some examples.



`http://www.lab4web.com/chelmiger/lyndyhop/lh_start.html`

or google “lyndyhop”

What about more complicated *L*-systems?

All the *L*-systems that we've seen so far draw single lines. But what if we want the lines to "branch"? We need another symbol:

[$\longrightarrow$ drop a "breadcrumb" that remembers where you are

] $\longrightarrow$ return to the last "breadcrumb" you dropped

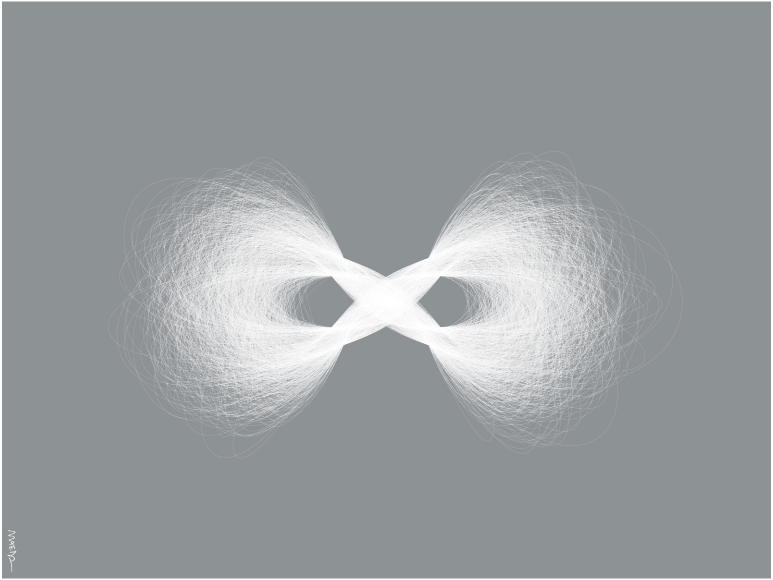
This lets us build some more complicated designs.

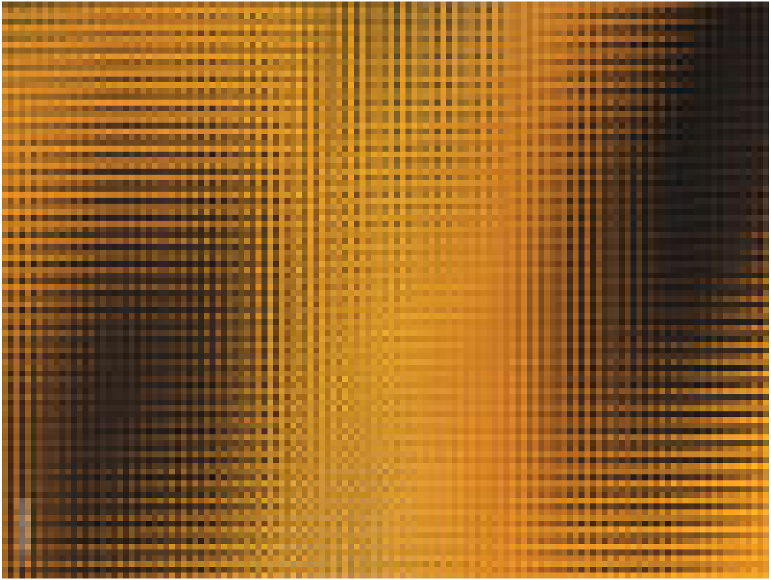
Outline

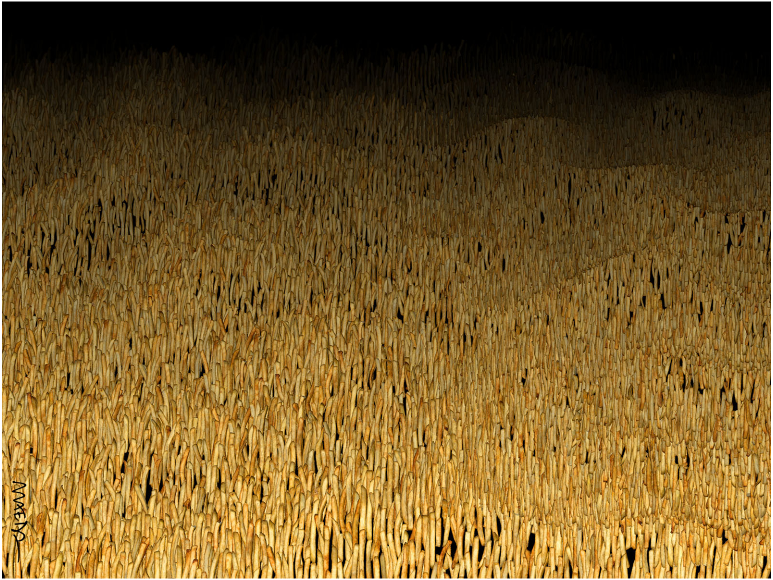
Using algorithms to create designs is an exciting idea which can generate some beautiful results. Here are some favorite images of mine which were created this way. Maybe they will inspire you to move in new directions. All of them come from the website of the graphic designer John Maeda.

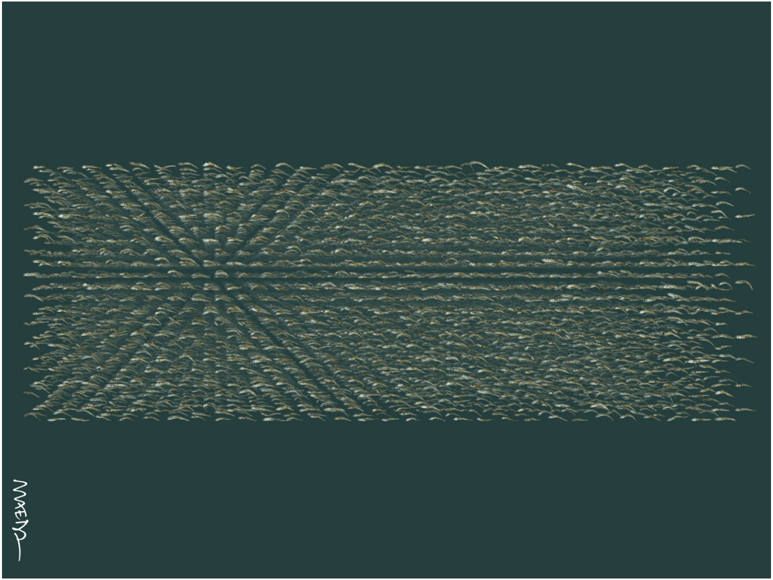
`www.maedastudio.com`

Math Butterflies









Thank you!

Thank you for having me speak!