Math 4500/6500 Minihomework: The trapezoid rule

This homework assignment covers our notes on the trapezoid rule and its error analysis.

1. Prove that if $\lambda_i \geq 0$ and $\sum_{i=1}^{n} \lambda_i = 1$, then for any sequence of numbers $x_1, \ldots, x_n$ we have

$$\min\{x_i\} \leq \sum_{i=1}^{n} \lambda_i x_i \leq \max\{x_i\}$$

2. Suppose that $f(x)$ is a concave down function, meaning that the graph of $f(x)$ lies above the chord connecting any two points on the graph. Prove that the trapezoid rule *under*estimates the integral of $f(x)$ over any interval.

3. We saw in the demonstration that the trapezoid rule works astonishingly well for the integral $\int_0^{4\pi} e^{\sin x}\, dx$. Compare the results from integrating via the trapezoid rule with 5, 10, and 15 intervals with the results produced by *Mathematica*'s built-in `NIntegrate` method. Which is better?