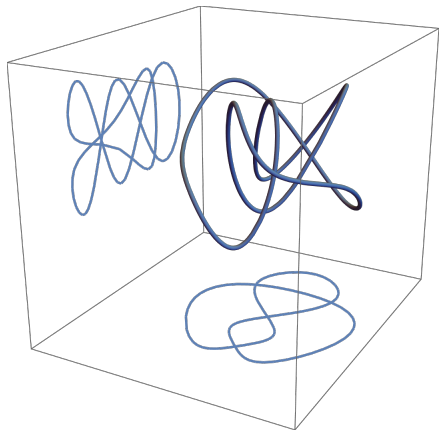
Math 4250 Minihomework: Integralgeometric measure

In this minihomework, we'll explore some consequences of our theorems on integralgeometric measure and on computing curve lengths by intersections. There are some physical activities to do and models to build and examine during this homework, so you might want to start a little earlier than usual. We first recall our notation and our theorem. As usual, we'll use len for the length of a curve. Given a unit vector $\vec{n} \in \mathbb{R}^3$, we'll let $P_{\vec{n}}$ denote projection to the plane though the origin with normal vector $\vec{n}$. The picture below shows a space curve and two of its projections (though we moved the planes away from the origin to avoid overlaps in the picture).



Our theorem on integralgeometric measure says that

Theorem 1 *Suppose that $\vec{\alpha} : [a, b] \to \mathbb{R}^3$ is a parametrized curve in $\mathbb{R}^3$. We have*

$$\mathrm{len}\,\vec{\alpha} = \frac{1}{\pi^2} \int_{\vec{n} \in S^2} \mathrm{len}\, P_{\vec{n}}\,\vec{\alpha}\, \mathrm{dArea}_{S^2}$$

1. (20 points) The theorem on integralgeometric measure says that if you know *all* of the lengths of projections of $\vec{\alpha}$ from all directions exactly, you can compute the length of $\vec{\alpha}$ exactly by integrating. But integrating over the sphere is the same as averaging over the sphere (up to dividing by the area of the sphere), so we can rewrite the theorem as

$$\mathrm{len}\,\vec{\alpha} = \frac{4}{\pi}\, \mathrm{mean}_{\vec{n} \in S^2}\, \mathrm{len}\, P_{\vec{n}}\,\vec{\alpha}$$

   We'll probably only have *some* projections and our measurements of the lengths of the projections will have some experimental error. Is the theorem still a useful way to estimate length? In this question, you'll explore this situation by experiment.

   Obtain a piece of wire[1] less than 1 foot long, and a ruler.[2]

[1] The wire needs to hold its shape. I find that pipe cleaner (available at craft supply stores or Walmart) or a wire laundry hanger both work pretty well. You can also get wire of this type from Amazon or a hobby store.

[2] You can print out a ruler if you don't own one.

(1) (5 points) Measure the wire using your ruler and record the measurement. Submit the measurement and take a photograph of the wire next to the ruler with your phone. Submit the image and the measurement.

(2) (5 points) Crumple and fold the wire into a 3 dimensional shape of your choice.[3]

Use your phone to photograph the crumpled wire from at least 6 directions of view. It helps if you stand a little ways away and zoom in, but it's not critical. Make sure that your wire is centered in the image and that the ruler is visible (and square-on to the camera) in the image. Submit all of your images.[4]

[3]This assignment will be harder if you squash the wire into a really small ball, so you might want to read the entire assignment before doing this step.

[4]Microsoft Word will allow you to drag and drop images into a text document and print out a single PDF if needed.

(3) (5 points) Measure the length of the wire in each image[5] and record your measurement. Submit a written description of your measurement process, the data, and annotated versions of your images showing how you measured.

[5] I like to use Fiji `https://imagej.net/Fiji` to do this, but there are plenty of other tools available. Note that you'll have to use the ruler in your image to get an answer with a consistent scale.

(4) (5 points) Average your 6 (or more) measurements and multiply by $\frac{4}{\pi}$ to get a prediction for the total length of your wire. How close is your prediction to the actual length of the wire? Write a few sentences about how you might improve the accuracy of your prediction.