# Math 4250 Minihomework: Constructing curves.

In this minihomework, we get to practice some of the calculus tricks that we used to solve the tractrix problem and to try our hand at constructing some other curves.

1. (25 points) (Hyperbolic trig functions) We are used to the usual trigonometric functions $\sin(x)$ and $\cos(x)$. We're now going to introduce two new functions: $\sinh(x)$ and $\cosh(x)$:

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}.$$

   (a) (5 points) Compute $\cosh 0$ and $\sinh 0$.

   (b) (5 points) Compute $\frac{d}{dx} \sinh x$ and $\frac{d}{dx} \cosh x$ in terms of $\sinh x$ and $\cosh x$. Compare to $\frac{d}{dx} \sin x$ and $\frac{d}{dx} \cos x$.

(c) (5 points) Simplify $\cosh^2 x - \sinh^2 x$ as much as possible. Show all work.

(d) (5 points) Use a computer to graph $\cosh x$ and $\sinh x$ for $x \in [0, 4]$.

(e) (5 points) Use a computer to plot the parametrized curves

$$\vec{\alpha}(t) = (\cos t, \sin t), t \in [-10, 10] \quad \vec{\beta}(t) = (\cosh t, \sinh t), t \in [-1.5, 1.5]$$

Can you recognize these curves? Why are $\cosh t$ and $\sinh t$ called hyperbolic trigonometric functions?

2. (20 points) Suppose that $f(x)$ is a function, and $g(x)$ is its inverse function; that is, that

$$f(g(x)) = x,$$

(a) (5 points) Differentiate both sides of the equation above with respect to $x$ and solve for $g'(x)$ to find a relationship between $g'$ and $f'$.

(b) (5 points) Recall that the inverses of the trig (and, as it turns out, the hyperbolic trig) functions are preceded by "arc"[1]. This means that

$$\sin(\arcsin x) = x, \quad \cos(\arccos x) = x,$$

We can compute the derivatives of the inverse trig functions by using the inverse function/chain rule trick from 2. For example:

$$\frac{d}{dx} \arcsin x = \frac{1}{(\sin'(\arcsin x)} = \frac{1}{\cos(\arcsin x)}.$$

At this point, we remember that $\cos^2 x + \sin^2 x = 1$, so $\cos x = \sqrt{1 - \sin^2 x}$ and so we can continue:

$$\begin{aligned} \frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1 - \sin^2(\arcsin x)}} \\ \frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1 - x^2}}. \end{aligned}$$

Repeat the above derivation to figure out $\frac{d}{dx} \arccos x$. What does the answer tell you about the functions $\arccos x$ and $\arcsin x$?

[1] Why? Well $\arcsin x$ is the angle $\theta$ for which $\sin\theta = x$. And if you measure $\theta$ in radians, the angle $\theta$ is the angle covered by an arc of length $\theta$ on the unit circle. So $\arcsin x$ literally means "the arc whose sine is $x$".

(c) (5 points) The hyperbolic trig functions also have inverses

$$\sinh(\operatorname{arcsinh} x) = x \quad \text{and} \quad \cosh(\operatorname{arccosh} x) = x$$

Compute $\frac{d}{dx} \operatorname{arcsinh} x$ and $\frac{d}{dx} \operatorname{arccosh} x$ using a similar method to the above. Is there a similar conclusion about the relationship between $\operatorname{arcsinh}$ and $\operatorname{arccosh}$?

(d) (5 points) Now integrate both sides of your answers in 2c to express these derivatives as integral formulae. Hang on to these, as you'll need them soon!