

Math 4500/6500 Homework #4

This homework assignment covers our notes on polynomial interpolation and interpolation errors. You are welcome to look at the code from the *Mathematica* notebooks, but when the problems say “write a piece of code to” they mean “write your own code from scratch”, not “modify the code in the notebook” or “find a piece of code on the web”.

1. Use the Lagrange process by hand to obtain a polynomial which interpolates the data

x		0	2	4	8
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y		7	11	28	63

Alternate: You are also welcome to implement the Lagrange process in *Mathematica*. It's up to you. Of course you should use *Mathematica's* `InterpolatingPolynomial` function to check your work, but calling `InterpolatingPolynomial` is *not* the same as implementing the Lagrange process.

2. Verify that the polynomials

$$p(x) = 3 + 2(x - 1) + 4(x - 1)(x + 2) \quad \text{and} \quad q(x) = 4x^2 + 6x - 7$$

both interpolate the data

x		1	-2	0
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y		3	-3	-7

(that is, $p(1) = q(1) = 3$ and so forth). Explain why this is not a violation of the uniqueness theorem for polynomial interpolation.

3. From census data, the population $p(y)$ of the United States (as a function of the year y) was given in millions of people by

y		1950	1960	1970	1980	1990
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p		150.7	179.3	203.3	226.5	249.6

Use Newton's interpolation process (by hand) to come up with a polynomial interpolating this data. Predict the population of the United States in 1920 and in 2000 using this model. What do you learn?

4. (Challenge) Use `CountryData` to repeat the above experiment for Kenya, using data from 1970, 1980, 1990, and 2000 to construct an interpolating polynomial predicting the country's population in 2010 and in 1930. Does this work any better?
5. Prove that the divided difference $f[x_0, x_1] = f'(c)$ for some $c \in [x_0, x_1]$ (again, this is to be done by hand).

6. Find the interpolating polynomial for the United States population data in the table in Question 3 using *Mathematica*'s function `InterpolatingPolynomial`.
7. Use *Mathematica* to find a polynomial of degree 10 which interpolates $f(x) = e^x$ on 11 equally spaced points in the interval $[0, 2]$. Plot this polynomial along with $f(x)$. (Challenge) Make an interactive version of this exercise where the user can vary the interpolation points using `Manipulate` and watch the change in the plot.
8. Use *Mathematica* to find a polynomial of degree 10 which interpolates $f(x) = e^x$ on 11 Chebyshev nodes distributed across the interval $[0, 2]$. Compare the results to $f(x)$ and to the polynomial from the last exercise.
9. Use our theorems on interpolation error to find a theoretical error bound for the polynomial of degree 10 interpolating $f(x) = e^x$ at 11 equally spaced points on $[0, 2]$ as an approximation to $f(x) = e^x$. Compare your theoretical bound to the actual error using *Mathematica*.