Math 4500/6500 Homework #2

This homework assignment covers our notes on *Mathematica* programming. The assignment should be turned in via email as a Mathematica notebook. Your notebook should include your name, should have a title and text blocks containing an explanation of what you're doing, and should run cleanly if I select and evaluate all cells. In order to help me grade the homework, it's really important that you name the file in a particular way. I'd like the filename to be:

`YourLastName_HW2_MATH4500_Year.nb`

If you have more than one file, name them `YourLastName_HW2_MATH4500_Year_a.nb`, `YourLastName_HW2_MATH4500_Year_b.nb`, etc. Please don't worry about it if you're actually in MATH6500, just name it 4500 anyway. This will help me keep track of which homework assignments I have and which I don't. Otherwise, I tend to end up with a folder of 30 files all named "Homework2.nb", which is a nightmare!

Part of the point of this homework assignment is to get you used to the process of acquiring the information you need in *Mathematica*. The "Help" menu will be your friend here, especially the Virtual Book and the Documentation Center.

1. Problems

1. Define functions for $f(x) = x^2 - 2x + 3$ and $g(x) = \frac{\sinh x}{2x}$. Plot each function individually on $[1, 3]$. Plot them together on $[1, 3]$. Label the axes of the graphs "Input" and "Yield".

2. Plot the function $f(x, y) = \sin xy$ on $[0, 10] \times [0, 10]$ as a surface and as a contour plot. Make the plot look as nice as you can.

3. Find the first 17 terms of the Taylor series for $e^{\sin x + 2x}$ centered at $x = 0$. Convert this Taylor series to a polynomial function and plot it and the original function on $[0, 4]$.

4. Find the first 5 terms of the Taylor series for $\sinh x$ centered at $x = 1$. Integrate this polynomial.

5. Find the the 5th derivative of $\cos(\sin x)$. Find the value of this derivative at $x = 3$ using a transformation rule.

6. Use `Cases` and a pattern to find all of the terms in the 6th derivative of $\cos(\sin x)$ which contain $\sin^n(\text{something})$ where $n > 1$. (Challenge) Use a pattern with alternatives to include terms which have $\sin(\text{something})$.

7. Use a transformation rule to turn every $\sin(\text{something})$ in the 6th derivative of $\cos(\sin x)$ into a $\sinh(\text{something})$. Be careful that you get them all!

8. Construct a list of the numbers from 1 to 10 using `Range`. Use `Map` and a function to construct a list of the squares of these numbers. Now find the sum of these squares. For extra style points, find the sum using the `@@` operator.

9. Newton's method constructs a sequence of points $x_0, x_1, x_2, \ldots$ which approximate the solution of $f(x) = 0$ by the rule

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

   Write this rule as a *Mathematica* function for $f(x) = \sin x$. Given x_0 and n, write a function that uses `Nest` to compute x_n. Now use the fact that $\sin \pi = 0$ to improve the approximation $x_0 = 3.0 \simeq \pi$ to a value for π given by x_n that is correct to 10 digits.

10. (Small Data) Load the file `adventures_of_sherlock_holmes.txt` from the course webpage at `http://www.jasoncantarella.com/downloads/adventures_of_sherlock_holmes.txt` into a single *Mathematica* string with the `Import` command. Use `StringSplit` and a string pattern to split the resulting string into a list of words. How many words are in the file? How many times does the word "Watson" appear in the text? How many times does the word "Holmes" appear in the text? How many times does the phrase "my dear Watson" occur in the text?

11. Produce an Association which counts the frequency of the various letters in *The Adventures of Sherlock Holmes* using `CharacterCounts`. Note that you'll either have to preprocess the string to remove non-letter characters (spaces, punctuation, newlines) before counting, or postprocess the output of `CharacterCounts` to remove the non-letter characters (look up `DropKey`).

12. Convert the Association above (which counts the number of times a letter occurs) to an Association which contains the relative probability of each letter in *The Adventures of Sherlock Holmes* by using `Map` and `Total`.