Math 4500/6500 Minihomework: The trapezoid rule

This homework assignment covers our notes on the trapezoid rule and its error analysis.

1. Prove that if $\lambda_i \geq 0$ and $\sum_{i=1}^n \lambda_i = 1$, then for any sequence of numbers x_1, \dots, x_n we have

$$\min\{x_i\} \le \sum_{i=1}^n \lambda_i x_i \le \max\{x_i\}$$

- 2. Suppose that f(x) is a concave down function, meaning that the graph of f(x) lies above the chord connecting any two points on the graph. Prove that the trapezoid rule *under* estimates the integral of f(x) over any interval.
- 3. We saw in the demonstration that the trapezoid rule works astonishingly well for the integral $\int_0^{4\pi} e^{\sin x} dx$. Compare the results from integrating via the trapezoid rule with 5, 10, and 15 intervals with the results produced by *Mathematica*'s built-in NIntegrate method. Which is better?