Math 4250/6250 Final Exam

This in-class exam covers material from the entire class. Please don't be afraid to read over the notes and the sections in the book as you work on the problems– there is more in the notes than we were able to cover in the lectures, and some of those extra facts might be helpful to you as you work on the exam problems.

You **are** permitted to use	You **are not** permitted to use
Your book (printed out or bound copy)	Technology of any kind
Printouts of course notes	Printouts not prepared by you
Any other material (notes) prepared by you	

This is a solo exam– please do not talk during the exam period. If you have a question, please raise your hand, remain seated, and I will come to you.

Hints on exam-writing

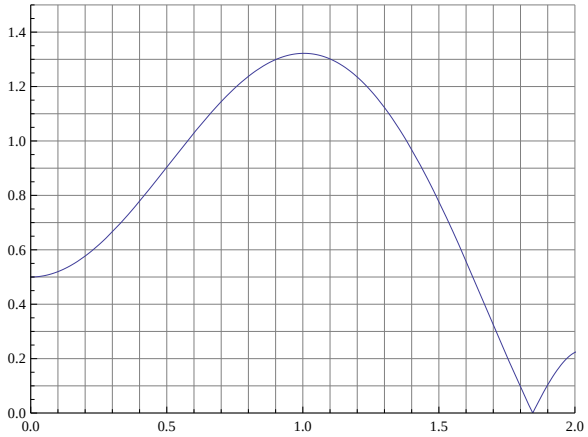
At this stage in your mathematical life, you are ready to learn to communicate your ideas to other people in a clear and organized way. When you work in industry or academia after graduation, a solution to a problem or a good idea that you can't convince others of will be worse than useless– even if you get the right answer! Nobody trusts a derivation that they can't understand.

So remember the following things when you write your exams:

(1) Mathematics is written in complete english sentences which explain not just what you are doing but why you are doing it. Every time you write down an equation, you should write a corresponding sentence which explains how you got there.
(2) There are few things more dangerous than a mathematical fact which you (think) you know, but can't find any reference for. Often these things are just wrong. And even when you are exactly right, when you are working with other people, nobody will believe you if you can't cite a reference. So it's very important to cite a specific page (either in your book or in the course notes) when you use a fact, formula, or definition.
(3) You are being graded not just on your solution but on how well you explain it. If I can't understand your solution after an honest attempt to follow your logic, you will fail the "explanation" portion of the exam and you will not get credit (even if the math is right!). This is exactly what happens in industry and research– you will get no credit with your boss or with your company for solving a problem if your coworkers can't easily understand and check your solution.

Please complete all problems.

1. Consider the curve
$$\alpha(t) = (t, \cosh t, 0)$$
Compute the curvature and torsion of $\alpha(t)$. It may help to remember the following facts about hyperbolic trig functions:
$$\frac{d}{dx}\cosh x = \sinh x, \quad \frac{d}{dx}\sinh x = \cosh h, \quad \cosh^2 x - \sinh^2 x = 1.$$
2. Use Crofton's formula to estimate the length of the curve in the picture below following the example on pages 45-46 of your book.



Please explain your work.

3. Suppose that $\alpha(v) = (0, y(v), z(v))$ is an arclength parametrized curve in the y-z plane. Consider the surface parametrized by
$$X(u, v) = (u, y(v), z(v)).$$
Compute the first fundamental form for this surface. Construct a map from this surface to the $x - y$ plane and show that this map is an isometry.
4. Name and sketch three points on everyday objects with positive Gauss curvature, three points with negative Gauss curvature, and three points with zero Gauss curvature.
5. Consider Enneper's surface
$$X(u, v) = \left(u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + vu^2, u^2 - v^2\right).$$
Show that
   (1) The first fundamental form is given by
   $$E = G = (1 + u^2 + v^2)^2, \quad F = 0.$$
   (2) The second fundamental form is given by
   $$e = 2, \quad g = -2, \quad f = 0.$$
   (3) The principal curvatures are
   $$k_1 = \frac{2}{(1 + u^2 + v^2)^2}, \quad k_2 = -\frac{2}{(1 + u^2 + v^2)^2}.$$
   (4) Compute the Gauss and Mean curvature of this surface.