Math 4220/6220 Exam #1

This exam covers Chapter 1 of our book. You are allowed to use:

- A linear algebra book of your choice.
- A multivariable calculus book of your choice.
- The first four chapters of Munkres' Topology book.
- MAPLE, Mathematica, Matlab, or any similar calculational aid
- your book
- your notes from class
- the notes posted on the course webpage.

Please don't use any other resources. The exam will be due next Saturday at 1pm. During the exam period, please refrain from discussing course material with the other students. You are welcome to ask me questions either by email or during class time (private questions don't seem entirely fair).

1. Consider the space of 6 sided polygons in $\mathbf{R}^2$, which we will denote $C_6(\mathbf{R}^2)$. Prove that this space is a differential manifold which is diffeomorphic to $\mathbf{R}^n$, and find n.
2. Let the space $\mathrm{Pol}_6(\mathbf{R}^2)$ denote the space of (closed) 6-sided polygons in $\mathbf{R}^2$. Prove that $\mathrm{Pol}_6(\mathbf{R}^2)$ is a smooth submanifold of $C_6(\mathbf{R}^2)$ and find its dimension and codimension. (Hint: Define a map from $C_5(\mathbf{R}^2) \to \mathbf{R}^2$ by taking the difference of the first and last vertices as a vector. Can you express Pol_6 as the inverse image of a regular value?)
3. Consider the space Tri of triangles in $\mathbf{R}^2$. We observed in class that this is a smooth manifold diffeomorphic to $\mathbf{R}^6$. Consider the map $f : \mathrm{Pol}_6 \to \mathrm{Tri}$ given by taking the midpoints of the line segments connecting the first, third, and fifth vertices. Prove or disprove: This map is a submersion.
4. Consider the map from $\mathrm{Tri} \to \mathrm{Pol}_6$ given by adding vertices at the midpoints of each side of the triangle. Prove or disprove: this map is an immersion.
5. (Bonus question). Prove that the space of quadrilaterals whose vertices lie on a common circle is a smooth submanifold of the space of quadrilaterals in $\mathbf{R}^2$. Find its dimension and codimension.