Math 3500 - Exam 2

April 8, 2003

There are 7 questions on this exam. You are to complete *five* of these questions. If you find time to work on more than five of the questions, please indicate which five you want graded. If you don't indicate, I'll pick randomly. Please work carefully, and *check your work where possible*. As usual, you are permitted one page of notes.

Good luck!

1. Compute the pullback $g^*(\omega)$ of the one-form $\omega = x^2 dx + y^2 dz$ on $\mathbf{R}^3$ to $\mathbf{R}^1$ under the map $g(t) = (\cos t, \sin t, t)$.
2. Compute the integral of $\omega = 2xydx + x^2 dy$ around the unit circle in $\mathbf{R}^2$. (*If* you decide to parametrize the circle and pull back to R^1, please use the parametrization $g(\theta) = (\cos\theta, \sin\theta)$.
3. Suppose that $\omega = 2xdx + 4xy^2 dy + dz$ is a 1-form in $\mathbf{R}^3$. Compute $d\omega$ (a 2-form in $\mathbf{R}^3$).
4. Find a potential function for the one-form $\omega = x\sin y dx + (e^y + \frac{1}{2}x^2\cos y)dy$ in $\mathbf{R}^2$ (that is, find a function f so that $df = \omega$) or prove that no such function exists. (Hint: This is not a trick question.)
5. Let g be the map $g(u,v) = (\cos u, \sin u, v)$, which parametrizes the cylinder $\{x^2 + y^2 = 1\}$ in $\mathbf{R}^3$. Let ω be the two-form $x^2 dx \wedge dz + ydx \wedge dy + e^z dy \wedge dz$. Compute $g^*(d\omega)$ by any means necessary. (Hint: This *is* a trick question.)
6. Compute the flux of the vector field $V(x,y,z) = (-y, x, 1)$ over the upper hemisphere of the unit sphere in $\mathbf{R}^3$.
7. Let ω be the 3-form $\omega = xw\sin z dx \wedge dy \wedge dw + e^w y^2 dy \wedge dz \wedge dw$ on $\mathbf{R}^4$ (where $\mathbf{R}^4$ has the coordinates x, y, z, and w.) Compute $d\omega$ and $d(d\omega)$.