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Math 3500 - First Midterm

1. a. $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ $\vec{y} = \begin{bmatrix} 4 \\ 1 \\ 2 \\ 3 \end{bmatrix}$. Compute $(\vec{x}+\vec{y})\cdot(\vec{x}-\vec{y})$ using any method you like.

* b. Suppose $\vec{x}, \vec{y} \in \mathbb{R}^n$ each contain the numbers $1, \ldots, n$ as coordinates, in some order (as in part a). Compute $(\vec{x}+\vec{y})\cdot(\vec{x}-\vec{y})$.

2. The linear transformation T_θ is defined by rotating the plane counterclockwise by θ radians. Find the standard matrix for T_θ.

3. The space of $n\times n$ matrices is the same as $\mathbb{R}^{(n^2)}$ (just write out all the entries as a long vector). Prove or disprove:

   The set of skew-symmetric $n\times n$ matrices is a subspace of $\mathbb{R}^{(n^2)}$.

Note: Problems with a star are a little harder!

4. Find a matrix A for which $A^{10} = I$, but $A^9 \neq I$. (Hint: Let A be a 2×2 matrix, so it represents a linear transformation of the plane).

*5. Suppose that $A^3 = O$. Prove that $I + A + A^2$ is an invertible matrix. (Find an explicit formula for $(I + A + A^2)^{-1}$.)

6. Define:

a) upper-triangular

b) main diagonal

c) linear map

d) cross product

Bonus question: Given a square $(n \times n)$ matrix A, Suppose $A\vec{x} \cdot A\vec{y} = \vec{x} \cdot \vec{y}$ for all $\vec{x}, \vec{y} \in \mathbb{R}^n$. Prove that $A^T = A^{-1}$.

Note: Problems with a star are a little harder!