Day 1. Math 3500.

Intro and mechanics. (10 min)

Integrated multivariable calc/linear algebra.
Advanced course, depends on reading/homework.

Read $\rightarrow$ Class (discuss harder points from section & applications) $\rightarrow$ Homework

Gradescope. JB22BP.

Bookings.

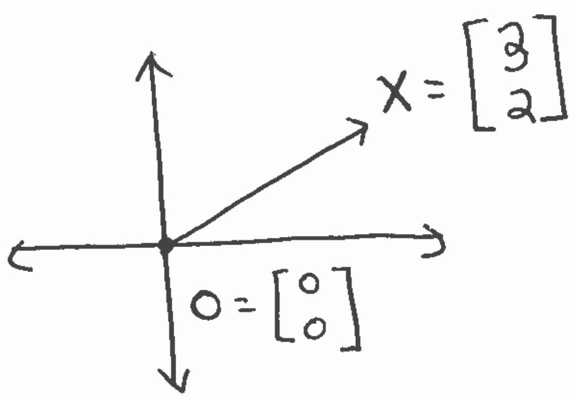
Diagnostic/Reading Quiz. ($\leq$ 10 min)

Definition. A vector $\mathbf{x} \in \mathbb{R}^n$ is an ordered list of numbers

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \end{bmatrix}$$

We can think of these as coordinates of a point in an n-dimensional space, and the vector as an arrow from the origin $\mathbf{0} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$ to $\mathbf{x}$.

Example.



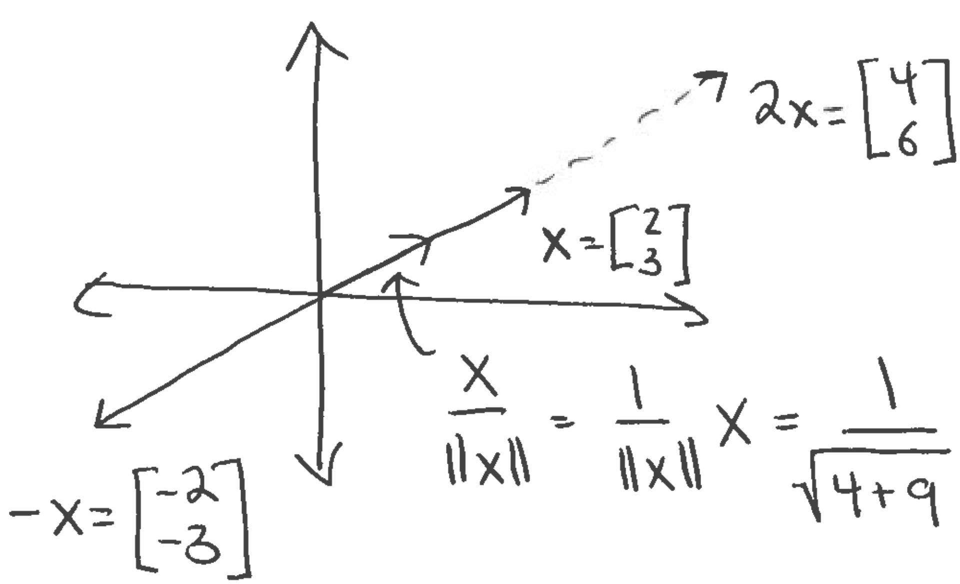
Definition. We <u>define</u> the length of $\mathbf{x}$ to be $\|\mathbf{x}\| = \sqrt{x_1^2 + \dots + x_n^2}$, and say that $\mathbf{x}$ is a <u>unit</u> vector if $\|\mathbf{x}\| = 1$.

Note. This definition implies the Pythagorean theorem. It's <u>not</u> the only useful definition of length for vectors.

Definition. If $c \in \mathbb{R}$ and $x \in \mathbb{R}^n$, we say c is a <u>scalar</u> and define the <u>scalar multiplication</u> of c and x by

$$c\mathbf{x} = \begin{bmatrix} cx_1 \\ \vdots \\ cx_n \end{bmatrix}$$

Example.



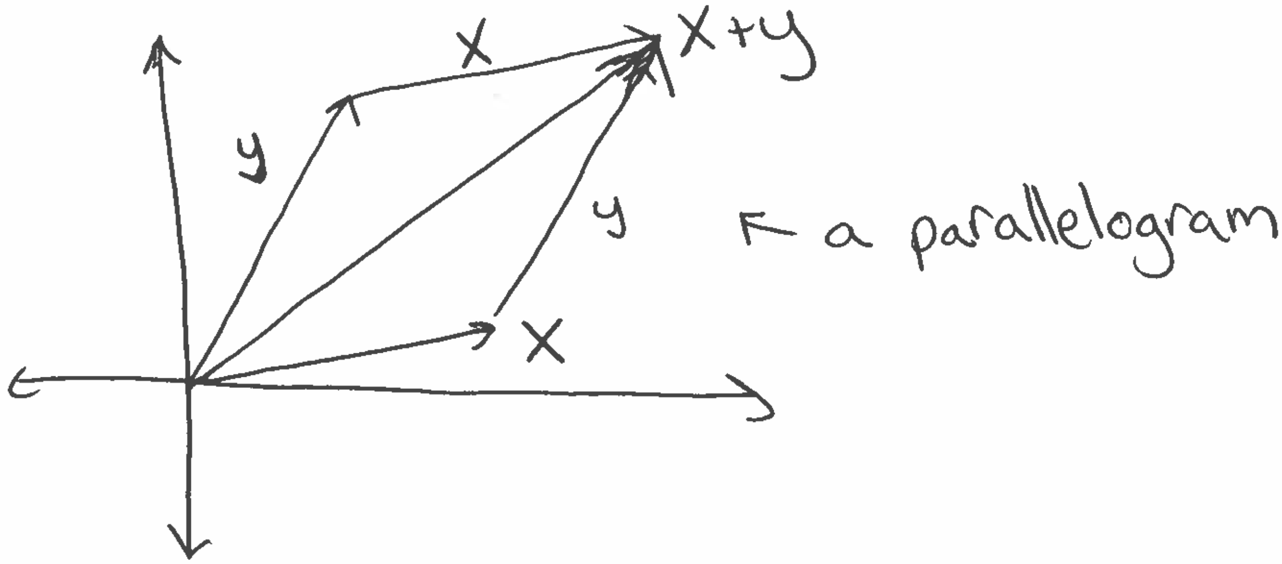
$$\frac{x}{\|x\|} = \frac{1}{\|x\|}x = \frac{1}{\sqrt{4+9}}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2/\sqrt{13} \\ 3/\sqrt{13} \end{bmatrix}$$

Definition. x and $y \in \mathbb{R}^n$ are <u>parallel</u> if there exists some c so $\mathbf{x} = c\mathbf{y}$.

Definition. If $x, y \in \mathbb{R}^n$ we define

$$x + y = \begin{bmatrix} x_1 + y_1 \\ \vdots \\ x_n + y_n \end{bmatrix}.$$

Example.



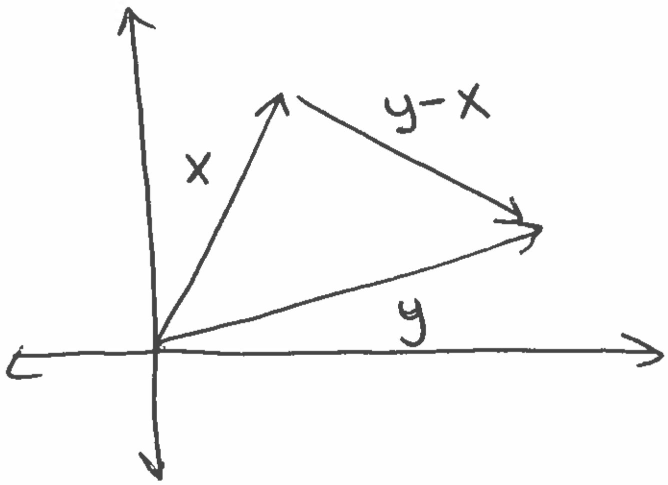
We call this the "parallelogram law" for vector addition: we add vectors by placing them "tail to head".

Lemma.

$$x + (-1)y = x - y = \begin{bmatrix} x_1 - y_1 \\ \vdots \\ x_n - y_n \end{bmatrix}$$

Why were vectors first important?
Geometry started with axioms from which propositions and theorems were derived by logical deduction. A great deal was accomplished this way, but the arguments require insight and creativity - they are also very long (the Pythagorean theorem is the 47th proposition in Euclid).
Vectors and coordinate geometry make many geometric proofs ~~pure~~ almost mechanical exercises in vector algebra.

If we think about



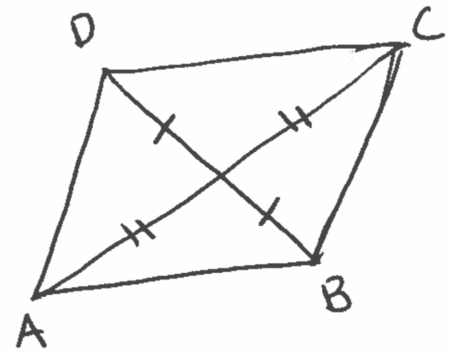
we see that the midpoint of the line segment is the point halfway from x to y, given by

$$x + \frac{1}{2}(y-x) = \frac{1}{2}y + \frac{1}{2}x$$

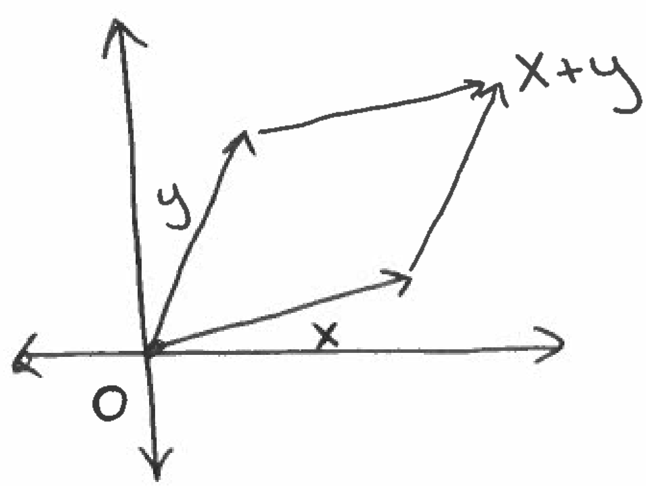
$$= \frac{1}{2}(x+y)$$

$$= m$$

Proposition. The diagonals of a parallelogram bisect each other.



Proof. We can write the parallelogram as



Now the diagonals join 0 to x+y, with midpoint

$$m_1 = \frac{1}{2}(0+(x+y)) = \frac{1}{2}x + \frac{1}{2}y$$

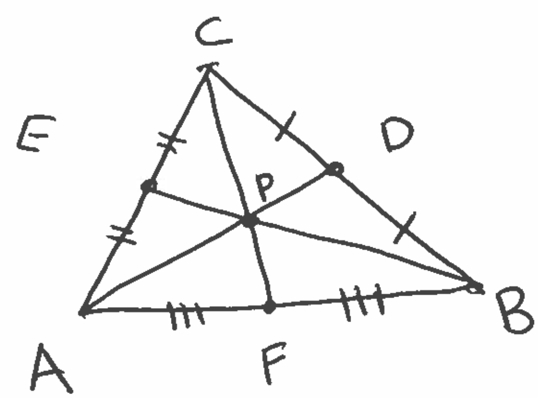
and x to y, with midpoint

$$m_2 = \frac{1}{2}(x+y) = \frac{1}{2}x + \frac{1}{2}y.$$

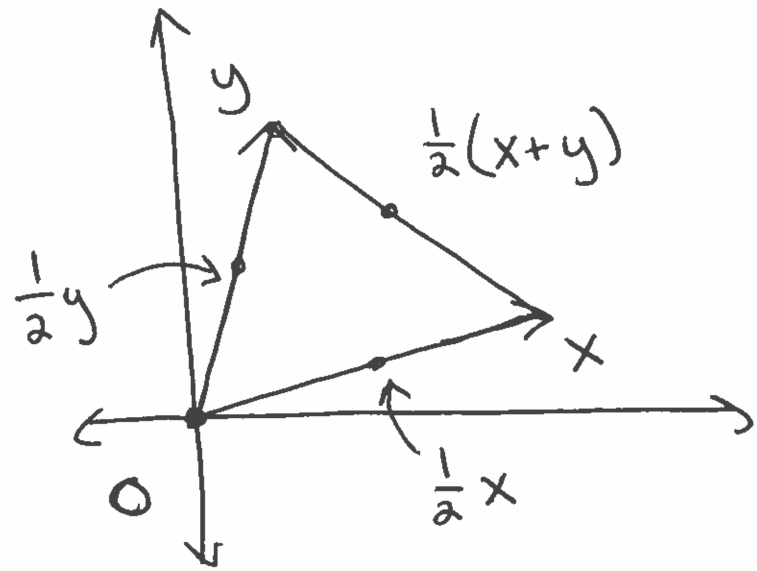
These are the same point. $\square$

Definition. The <u>median</u> of ΔABC through A is the line segment from A to the midpoint of BC. (other medians similar.)

Proposition. The three medians of ΔABC intersect at a single point p.



Proof. We may write $\triangle ABC$ as



We now construct the point $2/3$ of the way from each vertex to the opposite side's midpoint.

$$P_1 = 0 + \frac{2}{3}\left(\frac{1}{2}(x+y) - 0\right)$$
$$= \frac{1}{3}x + \frac{1}{3}y$$

$$P_2 = x + \frac{2}{3}\left(\frac{1}{2}y - x\right)$$
$$= x - \frac{2}{3}x + \frac{1}{3}y = \frac{1}{3}x + \frac{1}{3}y.$$

$$p_3 = y + \frac{2}{3}\left(\frac{1}{2}x - y\right)$$

$$= y - \frac{2}{3}y + \frac{1}{3}x = \frac{1}{3}x + \frac{1}{3}y.$$

Since these are all the same point p, we have completed the proof. $\square$