MATH 2260

Final Exam

December 12, 2007

NAME (please print legibly): ______________________________

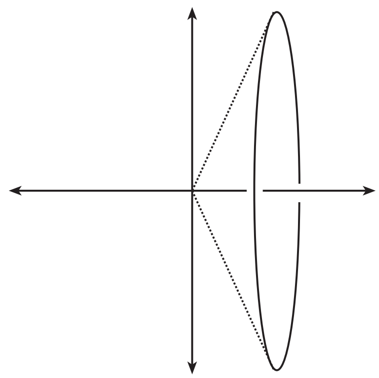
Your University ID Number: ______________________________

Please complete all questions in the space provided. Draw a box around your final answer. You may use the backs of the pages for extra space, or ask me for more paper if needed. Work carefully, and neatly (part of your grade will be based on how well your work is presented).

Try to complete the problems you find easier before going back to the harder ones. In particular, the last two questions on the exam are very easy vector questions, and you should make sure to answer them early on in case you run out of time later.
Good luck!

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
TOTAL	120	

1. (10 points) A cone is created by rotating the line $y = 2x$ around the x axis. Find the **surface area** of the portion of the cone with $0 \leq x \leq 4$. Do not include the circular "cap" on the end of the cone.



ANSWER: ____________________________

2. (10 points) The region between the curves $y = x + 1$ and $y = (x - 1)^2$ is rotated around the x axis. Sketch a graph of the two functions, find where they intersect, and compute the volume of the resulting solid of revolution **using the washer method**.

ANSWER: ____________________________

3. (10 points) The region between the curves $y = x^2$ and $y = x^3$ for $0 \leq x \leq 1$ is rotated around the x-axis. Sketch a graph of the two functions, and compute the volume of the resulting solid of revolution **using the method of cylindrical shells**.

ANSWER: ______________________

4. (10 points) Please match each rational function on the left hand side with the corresponding partial fraction decomposition on the right hand side.

1.

$$\frac{x^2+4x+12}{(x+2)(x^2+4)}$$

2.

$$\frac{x^2-4x+8}{(x-1)^2(x-2)^2}$$

3.

$$\frac{2x^2+8x+24}{(x+2)^2(x^2+4)}$$

4.

$$\frac{x^4-4x+8}{(x+2)(x^2+4)}$$

1.

$$x-2+\frac{4}{x+2}-\frac{4x-4}{x^2+4}$$

2.

$$\frac{-8}{x-2}+\frac{4}{(x-2)^2}+\frac{8}{x-1}+\frac{5}{(x-1)^2}$$

3.

$$\frac{1}{x+2}+\frac{2}{(x+2)^2}+\frac{-x+2}{x^2+4}$$

4.

$$\frac{1}{x+2}+\frac{4}{x^2+4}$$

ANSWER: ____________________________

5. (10 points) The length of the orbit of the planet Pluto in astronomical units (AU) is (really!) given by the integral

$$L = 4a \int_0^{\pi/2} \sqrt{1 - e^2 \sin^2 x}\,\mathrm{d}x.$$

where $a = 39.78$, $e = 0.2529$. Use Simpson's rule with $n = 6$ to approximate this integral numerically. (By the way, 1 AU is 149.6 million km, in case you were curious.)

ANSWER: ____________________

6. (10 points) Evaluate the integral

$$\int \arcsin x \, \mathrm{d}x.$$

Hint: Integration by parts.

ANSWER: ______________________________

7. (10 points) Does the series

$$\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$$

converge or diverge? Use any test you like to answer the question.

ANSWER: ____________________________

8. (10 points) Derive the first 5 terms of the Taylor expansion for the function $f(x) = \sqrt{1+x}$ around the point $a = 0$. (Hint: Use the definition of Taylor series and compute derivatives.)

ANSWER: ____________________________

9. (10 points) Use Taylor series to estimate the integral

$$\int_0^3 e^{-x^2}\,\mathrm{d}x$$

by writing down a Taylor series for e^{-x^2} and integrating it, then evaluating the resulting series at $x = 0$ and $x = 3$. (Use at least 5 terms in your Taylor series.)

ANSWER: ______________________

10. (10 points) Find the first five nonzero terms in the Taylor series **for the derivative** of the function

$$f(x) = \frac{e^x}{1-x}.$$

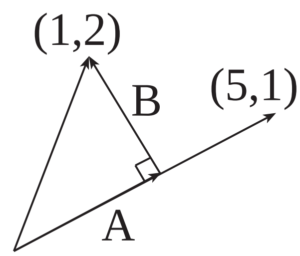
at the point $a = 0$. (Hint: It is probably not best to try to take five derivatives of this function ... can you find another way to compute the Taylor series?)

ANSWER: ____________________

11. (10 points) Calculate the length of the vector $(1, 4, 5)$ and the angle between the vectors $(1, 2, 2)$ and $(3, 2, 1)$.

ANSWER: ____________________________

12. (10 points) The vector $(1,2)$ is projected onto the vector $(5,1)$, resulting in a parallel vector (labelled A) and a perpendicular vector (labelled B). Find coordinates for A and B.



ANSWER: ____________________________