MATH 2260

Midterm Exam I

September 24, 2005

NAME (please print legibly): _____

Your University ID Number: _____

Please complete all questions in the space provided. Draw a box around your final answer. You may use the backs of the pages for extra space, or ask me for more paper if needed. Work carefully, and neatly (part of your grade will be based on how well your work is presented).

Try to complete the problems you find easier before going back to the harder ones. Good luck!

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	2	
10	10	
11	2	
12	10	
TOTAL	104	

1. (10 points) X-ray measurements reveal that an irregularly shaped solid is $15\,\mathrm{cm}$ tall and that the area of the cross section at height $x\,\mathrm{cm}$ is given by $(x^2 + 4x^3)\,\mathrm{cm}^2$. Find the volume of the solid in cm^3.

2. (10 points) Compute the volume of the solid created by revolving the region bounded by the curves $y = \cos x$, $y = \sin x$, $x = 0$ and $x = \pi/4$ around the x-axis. Use any method that you like.

3. (10 points) Find the length of the portion of the curve

$$x(t) = \ln t - \frac{t^2}{8}, \quad y(t) = t.$$

with t between 1 and 4.

4. (10 points) The portion of the curve $y = \sqrt{4x + 4}$ with x between 0 and 1 is rotated around the x axis to generate a surface of revolution. Find the area of this surface.

5. (10 points) Assume that the rate of change of atmospheric pressure $P(h)$ with respect to height h is proportional to $P(h)$. (Note: This is true if temperature is constant).

If the pressure at sea level ($h = 0$) is $101.3\,\mathrm{Kpa}$ and the pressure at $1000\,\mathrm{m}$ above sea level is $87.14\,\mathrm{Kpa}$, what is the pressure at the top of Denali ($6187\,\mathrm{m}$ above sea level)?

6. (10 points) Find the volume of the solid generated by revolving the area bounded by $y = e^{2x}$, $y = 0$, $x = 0$ and $x = 1$ around the y-axis.

7. (10 points) State the integration by parts formula and use it to show that

$$\int (\ln x)^n \, dx = x(\ln x)^n - n \int (\ln x)^{n-1} \, dx.$$

8. (10 points) The force of gravity (in Newtons) on a communications satellite is proportional to the inverse of the square of the distance r (in meters) from the satellite to the center of the earth. If we assume that the radius of the earth is $6400\,\mathrm{km}$, the constant of proportionality K is approximately 4×10^{17}.

Calculate the total work required to lift the satellite to geosynchronous orbit ($42,000\,\mathrm{km}$ above *sea level*). Keep in mind that the force required to lift the satellite is **changing** as the distance r increases.

Hint: Be careful of your units.

9. (2 points) Bonus: What is the total work required to raise the satellite to an infinite height above the earth?

ANSWER: _____

10. (10 points) Find the volume of the solid created by rotating the region bounded by the curve $y = 1/x$, the line $x = 1$ and the line $x = 100$ around the x axis.

11. (2 points) Bonus: What is the total volume of the (infinite) portion of the solid to the right of the line $x = 1$?

ANSWER: _____

12. (10 points) Find the **area** of the surface created by rotating the portion of the curve

$$y = 1 - x^2$$

with x between 0 and 1 around the y-axis.