MATH 2260

Final Exam

May 4, 2015

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Please complete all questions in the space provided. Draw a box around your final answer. You may use the backs of the pages for extra space, or ask me for more paper if needed. Work carefully, and neatly. **You must use words for each problem (preferably a short sentence, at least), to receive full credit.**

The questions are not in order of difficulty, so a good plan is to **skip over any questions that you find hard** and return to them after you've completed the easy questions.
Good luck!

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	35	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
TOTAL	155	

Part I: Chapters 6 (Applications of Definite Integrals) and 8 (Methods of Integration)

1. (10 points) Explain in your own words the difference between the **disk method** and the **cylindrical shell method** for computing the volume of a solid of revolution. Draw pictures of the same volume being computed by disks and by shells.

ANSWER: ______________________

2. (10 points) Find the integral

$$\int \sec^2 x \tan^2 x \, dx.$$

ANSWER: ____________________

3. (10 points) Find the integral

$$\int \frac{1}{x^3 + x}\, dx.$$

ANSWER: ____________________________

4. (35 points) (5pts) The formula for Simpson's rule is

$$\frac{h}{3}\big(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 4f(x_{n-1}) + f(x_n)\big).$$

Explain briefly what h and $x_0, x_1, \ldots, x_n$ are and what Simpson's rule is for.

ANSWER: ____________________________

(5pts) The error formula for Simpson's rule is

$$E < \frac{M(b-a)^5}{n^4}$$

Explain briefly what E, M and n are.

ANSWER: ____________________________

(10 pts) The following is actual data[1] for the **rate** $r(t)$ of total electric power consumption for the state of California on April 28, 2015.

Time (hours since midnight)	0	6	12	18	24
Consumption rate $r(t)$ (gigawatts)	24.5	23	29	31.5	25.0

The average rate of power consumption for the day is given by

$$R = \frac{1}{24}\int_0^{24} r(t)\,dt$$

Estimate the average rate of power consumption on April 28, 2015 using Simpson's rule.

(more space to work on the next page)

[1] Source: California Independent System Operator website.

(continued)

ANSWER: ______________________

(10 points) Historical data shows that the first 5 derivatives of $r(t)$ are always less than the bounds

Derivative	$r'(t)$	$r''(t)$	$r'''(t)$	$r^{(4)}(t)$	$r^{(5)}(t)$
Bound	< 20	< 22	< 18	< 6	< 4

Find a bound for the error in your estimate above.

ANSWER: ____________________________

(5 pts) One gigawatt is a power consumption rate of 1 billion joules (1 gigajoule) of electrical energy **per second**. Give an estimate (with error) in the form $A \pm B$, with units, for the total number of joules of electrical energy required to power California on April 28.

ANSWER: ____________________

Bonus (5pts) : If we could convert matter directly into electrical energy using $E = mc^2$, one Snickers bar would generate $\sim 5.7 \times 10^{14}$ joules of electrical energy[2]. We'll call this unit "the Snickers". Convert your estimate above into Snickers.

ANSWER: ____________________

[2] Assuming roughly 10% efficiency.

5. (10 points) State in your own words the definition of a geometric series. Give an example of a geometric series which converges. What number does your series converge to?

ANSWER: ______________________

6. (10 points) Use **any test or tests that you like** to determine whether the series

$$\sum_{n=1}^{\infty} \frac{e^n}{1+e^{2n}}$$

converges or diverges. (Note: A guess with written explanation may be worth as many as 4 points. A guess with no explanation gets no credit.)

ANSWER: ____________________

7. (10 points) Find the Taylor series for

$$f(x) = \frac{2+x}{1-x}$$

centered at 0.

Note: This question is harder than it looks, since you have to find the **whole** series (not just the first few terms). You might start by finding the Taylor series for $g(x) = \frac{1}{1-x}$.

ANSWER: ____________________________

8. (10 points) Use Taylor series to find

$$\lim_{x \to 0} \frac{\arctan x - \sin x}{x^3}.$$

Note: It is theoretically possible to do this directly with L'Hôspital's rule instead of using Taylor series (just differentiate the numerator and denominator enough times). However, it's really too lengthy for a test question, and it's a Calculus I problem. Therefore, to keep you from wasting time on that, **I will not give credit for that answer**.

ANSWER: ____________________________

9. (10 points) Find the angle between the vectors $\vec{u} = (1, -3, 4, 7)$ and $\vec{v} = (2, 6, -5, 1)$.

ANSWER: ________________________

10. (10 points) Find the area of the triangle whose sides are given by the vectors $\vec{u} = (1, 3, 5)$ and $\vec{v} = (0, -1, 7)$ (the three vertices of the triangle are $\vec{u}$, $\vec{v}$, and $\vec{0}$).

ANSWER: ______________________________

11. (10 points) (5pts) Find the length of the vector $\vec{u} = (1, -2, 4, 1)$.

ANSWER: ____________________

(5pts) Find a vector $\vec{v}$ so that $\vec{v} = s\vec{u}$ and $|\vec{v}| = 3$.

ANSWER: ____________________

12. (10 points) Suppose that $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^3$. Consider

$$(\vec{u} \times 5\vec{u}) \times \vec{v}$$

Is this expression a vector or a scalar (number)? Which vector or scalar is it and why? (The answer does not depend on $\vec{u}$ and $\vec{v}$.)

ANSWER: ____________________________

13. (10 points) Bonus question (seriously!)

Suppose that the probability that a new MacBook air is free from manufacturing defects is p and the probability that it is defective is $q = 1 - p$. The probability $g(n)$ that the n-th MacBook is the first defective one is

$$g(n) = p^{n-1}q.$$

The expected number of MacBooks inspected between defective MacBooks is

$$E = \sum_{n=1}^{\infty} n g(n) = \sum_{n=1}^{\infty} n\, p^{n-1}\, q$$

Use the theory of Taylor series to evaluate this sum (the answer will be in terms of p and q).
Hint: Consider the Taylor series for $\frac{1}{(1-x)^2}$.

(more space to think about the bonus question)