MATH 2260

Midterm Exam II

April 16, 2015

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Please complete all questions in the space provided. Draw a box around your final answer. You may use the backs of the pages for extra space, or ask me for more paper if needed. Work carefully, and neatly: **part of your grade will be based on how well your work is presented**.

Try to complete the problems you find easier before going back to the harder ones. Good luck!

QUESTION	VALUE	SCORE
1	10	
2	10	
3	15	
4	10	
5	10	
6	10	
7	10	
8	10	
TOTAL	85	

1. (10 points) What is the difference between a sequence and a series?

2. (10 points) Does the **series**

$$\sum_{n=1}^{\infty} \frac{\ln n}{n}$$

converge or diverge? Use any test you like to justify your answer. If you can't get a test to work, a correct guess is worth 2 points, and a guess supported by a few sentences describing your thought process may be worth as many as 4 points.

3. (15 points) Consider the **power series**

$$f(x) = \sum_{n=0}^{\infty} (-1)^n x^{2n}.$$

This problem has three parts:

1. Write out the first 5 nonzero terms of the series (3 points).

2. Find the values of x for which the series converges (6 points).

3. Find a formula (in terms of x) for $f(x)$ which is valid when the series converges (6 points).

4. (10 points) Consider the (convergent) alternating series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = L$$

This problem has three parts:

- (3 points) Write out the first 5 terms of the series.

- (6 points) Find n so that the partial sum s_n of the series is within 0.1 of L. Explain why your n works. Compute s_n.

(problem 5 continues on the next page)

(problem 5, continued)

- (6 points) Find an exact formula for L using the theory of Taylor series. Verify, using your calculator, that the partial sum you computed above is actually within 0.1 of L.

5. (10 points) Find the fourth **order** Taylor polynomial $T_4(x)$ for $f(x) = e^x \sin x$ centered at 0.

6. (10 points) It turns out to be the case that

$$\arcsin(x) = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \ldots$$

for x near zero. This question has two parts:

1. (5 points) Find the first three (nonzero) terms of the Taylor series for $\int \arcsin(x)\,dx$.

2. (5 points) Give the best numerical estimate you can for $\int_0^{\frac{1}{2}} \arcsin(x)\,dx$ as a sum of fractions.

3. (5 points) Discuss the error in your estimate above. How would you bound it?

7. (10 points) Does the series

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

converge or diverge? Use any method you like, but thinking is better than calculating. If you can't get a test to work, a correct guess is worth 2 points, and a guess supported by a few sentences describing your thought process may be worth as many as 4 points.

8. (10 points) Bonus question

Suppose that the probability that a new MacBook air is free from manufacturing defects is p and the probability that it is defective is $q = 1 - p$. The probability $g(n)$ that the n-th MacBook is the first defective one is

$$g(n) = p^{n-1}q.$$

The expected number of MacBooks inspected between defective MacBooks is

$$E = \sum_{n=1}^{\infty} ng(n) = \sum_{n=1}^{\infty} n\, p^{n-1}\, q$$

Use the theory of Taylor series to evaluate this sum (the answer will be in terms of p and q).
Hint: Consider the Taylor series for $\frac{1}{(1-x)^2}$.

(more space to think about the bonus question)