Subspaces.

Definition. A subset $V \subset \mathbb{R}^n$ is a <u>subspace</u> if

1) $0 \in V$
2) If $v \in V$ and $c \in \mathbb{R}$, $cv \in V$
3) If $v, w \in V$ then $v + w \in V$.

Examples.

$\{0\}$, $\mathbb{R}^n$, scalar multiples of $v \neq 0$
(line)

all sums of two nonparallel v, w

orthogonal complement of v

Nonexamples.

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ s.t. $x_1 + x_2 = 5$.

Definition. If $v_1, \ldots, v_k \in \mathbb{R}^n$ and $c_1, \ldots, c_k \in \mathbb{R}$, we say

$$c_1 v_1 + \ldots + c_k v_k = v$$

is a <u>linear combination</u> of $v_1, \ldots, v_k$.

The set of all linear combinations of $v_1, \ldots, v_k$ is called the <u>span</u>, and denoted $\mathrm{Span}(v_1, \ldots, v_k)$.

Example. We let $e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \ldots, e_n = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$.

Then $\mathbb{R}^n = \mathrm{Span}(e_1, \ldots, e_n)$.

These are called "standard basis" vectors.

Examples (if time allows)

Definition. Let $V, W \subset \mathbb{R}^n$ be subspaces. We say they are <u>orthogonal</u> if for every $v \in V$, $w \in W$ we have $v \cdot w = 0$.

Definition. Given a subspace $V \subset \mathbb{R}^n$, we define the <u>orthogonal complement</u> $V^\perp$ by

"perp"

$$w \in V^\perp \iff w \cdot v = 0 \text{ for all } v \in V.$$

Example.

$$\mathrm{Span}(e_1, e_2) = \mathrm{Span}(e_3)^\perp$$