Math 2250 Homework #1

This homework assignment covers three problems in the Interception lab exercise.

1. Problems

1. Plotting points on the video footage of the incoming ball tells us that the location of the incoming ball is given by the following data:

Time	Position (x,y)
5	(69.5, 147.6)
7	(109., 204.6)
9	(150.5,249.2)
11	(192., 278.6)
13	(233., 294.2)
15	(273.5, 294.2)
18	(332.5, 264.6)
21	(388., 206.6)
23	(420.5, 152.6)
25	(451.5, 89.6)

We know that the acceleration in the x-direction is zero and that the acceleration in the y direction (due to gravity) is constant. Notice that the units here are pixels per frame, so there is no reason to believe that the acceleration due to gravity is -9.8.

Fit a linear function $x1(t) = pt + q$ to the x data above and a quadratic function $y(t) = pt^2 + qt + r$ to the y data above. This function predicts the position of the incoming tennis ball as a function of time.

Note: The answers the class arrived at during Thursday's lab was

$$x(t) = -22.5399 + 19.3565t, \qquad y(t) = -54.4706 + 49.432t - 1.752t^2.$$

2. Suppose that we launch an intercepting tennis ball with velocity 75 from position (640,480) at time t_0. Find the horizontal position $X(t)$ and vertical position $Y(t)$ for this tennis ball in terms of the launch time t_0.

Note: The answer we arrived at in class on Monday was

$$X(t) = -64.952t + (640 + 64.952t_0).$$
$$Y(t) = -1.752t^2 + (-37.5 + 3.54t_0)t + (-1.752t_0^2 + 37.5t_0 + 480).$$

3. Suppose that the two tennis balls strike each other at time t_1. We know that at this time,

$$x(t_1) = X(t_1), \qquad y(t_1) = Y(t_1).$$

Since the equations $X(t)$ and $Y(t)$ for the intercepting tennis ball depend on t_0, this is a system of two equations in two unknowns (t_0 and t_1). Solve for the launch time (t_0) and resulting interception time (t_1) using any method you like. At what position (x, y) does the interception occur?

Note: We must solve the simultaneous equations

$$-22.5399 + 19.3565t_1 = -64.952t_1 + (640 + 64.952t_0).$$

$$-54.4706 + 49.432t_1 - 1.752t_1^2 = -1.752t_1^2 + (-37.5 + 3.54t_0)t_1 + (-1.752t_0^2 + 37.5t_0 + 480).$$

If we solve the first equation for t_1 (in terms of t_0) we get

$$t1 = 0.0118612(662.54 + 64.952t_0)$$

Plugging this into the second equation, we can rewrite that as an equation in t_0:

$$148.686 + 1.65401t_0 - 0.975247t_0^2 = 0$$

and solving this with the quadratic formula gives us $t_0 = -11.526$ or $t_0 = 13.224$ as solutions.