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Polygonal chains cannot lock in 4D. (English summary)
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A polygonal chain $P=(v_0,v_1,\dots,v_n)$ is a sequence of consecutively joined segments $s_i=v_iv_{i+1}$ of fixed lengths $l_i=|s_i|$ embedded in Euclidean space E^d. A polygonal tree is defined in an analogous manner.

The paper under review deals with the question whether an open simple polygonal chain or a simple polygonal tree can be straightened, i.e. stretched out in a straight line without violating simplicity. It is shown that every simple open chain and every simple tree in 4D can be straightened, by an algorithm that runs in $O(n^2)$ time and $O(n)$ space, and which accomplishes the straightening in $O(n)$ moves. Furthermore it is shown that every simple closed chain in 4D can be convexified, by an algorithm that runs in $O(n^6\log n)$ time and which accomplishes the straightening in $O(n^6)$ moves. All these results easily extend to higher dimensions.

The algorithm for straightening open chains in 4D works generically as follows: Move the first vertex v_0 until the segments s_0 and s_1 together form a straight segment and then proceed with the chain $P'=(v_0,v_2,\dots,v_n)$. The vertex v_0 is moved in a 3-sphere. The requirement that during the movement of v_0 the chain remains simple yields to a 1-dimensional obstruction diagram. The complement of this obstruction diagram in the 3-sphere is connected, therefore straightening is possible. In the nongeneric case, i.e. the desired position for v_0 lies on the obstruction diagram, one first has to move v_1 slightly. A careful motion planning gives the bound for the complexity of this algorithm.

Reviewed by *Ludwig Balke*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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