MATH 2250

Midterm Exam II – Special Robot-Free Edition! (But with lots of magnets.)

April 21, 2011

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Please complete all questions in the space provided. You may use the backs of the pages for extra space, or ask me for more paper if needed.

YOU ARE BEING GRADED ON EXPOSITION AS WELL AS CORRECTNESS.

Please do **NOT** simplify your answers (looking at them in "raw" form helps me understand your thought process). Work carefully, and try to complete the problems you find easier before going back to the harder ones. Good luck!

QUESTION	VALUE	SCORE
1	15	
2	10	
3	15	
4	10	
5	15	
6	10	
7	10	
8	5	
TOTAL	90	

1. (15 points) Find the derivative of the functions below:

- $f(x) = \sec(\ln x)$.

ANSWER: ____________________

- $f(x) = \arcsin(4x + 2)$.

ANSWER: ____________________

- $f(x) = 10^{x+3}$.

ANSWER: ____________________

2. (10 points) Find the maximum and minimum value of $f(x) = 4\cos(x^2 - 1)$ on the interval $[-1/2, 1/2]$.

ANSWER: ____________________

3. (15 points) The magnetic field B generated by an electromagnet obeys the equation:

$$I = B\left(\frac{K_1}{\mu} + K_2\right).$$

where I is the power input to the magnet, μ is a variable[1], and K_1 and K_2 are constants describing the geometry of the magnet.

Suppose that as we steadily increase the power I we reach a time when $\frac{dB}{dt} = 0$. Use implicit differentiation to solve for $\frac{d\mu}{dt}$ at this time. Your answer should involve $\frac{dI}{dt}$ as well as other variables.

ANSWER: ____________________

[1] Actually, μ describes how the material in the core of the magnet reacts to the magnetic field. Not that it matters.

4. (10 points) Suppose that we know that the force $f(I)$ produced by an electromagnet as a function of the power input I has the following values and derivatives for various I:

I	$f(I)$	$f'(I)$	$f''(I)$
2.	25.0489	13.0485	0.0234664
4.	51.191	13.0923	0.0199617
6.	77.4122	13.1271	0.014503
8.	103.691	13.1494	0.00762469

Use linear approximation to estimate $f(5.6)$. (For 3 bonus points, use *quadratic* approximation to estimate $f(5.6)$, too.)

ANSWER: ____________________

Now suppose the input power is supplied by a transformer which provides 4 ± 0.01 units of power. Use linear approximation to estimate the range of forces produced by the magnet as the power varies. (This is an error question.)

ANSWER: ____________________

5. (15 points) Find the (indefinite) integrals

- $\int x^3 + 5x^2 + 7dx$

ANSWER: ____________________

- $\int \ln(\cos(x)) \sin x dx$

ANSWER: ____________________

- $\int \frac{1}{\sqrt{6-4x^2}} dx$

ANSWER: ____________________

6. (10 points) Use the second derivative test to decide whether the critical point of

$$f(x) = \ln\left(\frac{1}{1+x^2}\right)$$

is a local MAX or MIN. What is the value of the function f at the critical point?

ANSWER: ____________________________

7. (10 points) Use L'Hôspital's rule to find the limit:

$$\lim_{x \to 0} \frac{\tan x}{\ln(1+x)}$$

ANSWER: ________________________

8. (5 points) Bonus question! Find the derivative of

$$f(x) = x^{x^x}$$

There is no need to simplify your answer.

ANSWER: ________________________