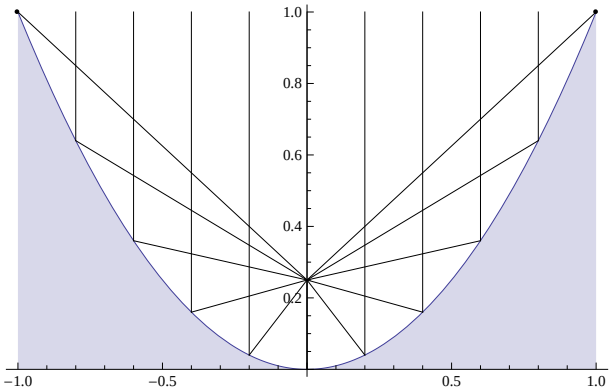
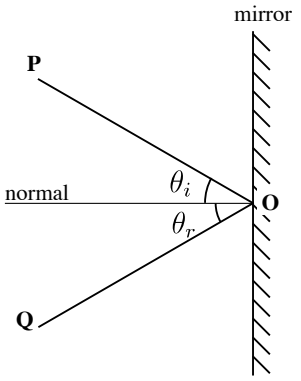
Math 2250 Lab #3 : Solar Collector

The assignment for this lab is to use our knowledge of tangent lines to design a solar collector. The collector should be a curved mirror defined by an equation $y = f(x)$ which focuses (vertical) sunbeams striking the mirror on a collector mounted at the point $(0, h)$ as shown in the picture below left. The first thing we need to know is how light bounces off of a mirror. That is given by the *law of reflection*: angle of incidence equals angle of reflection. This is summed up in the picture below right.

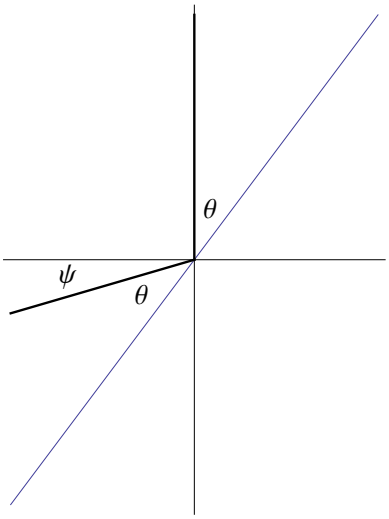


A solar collector focusing sunbeams



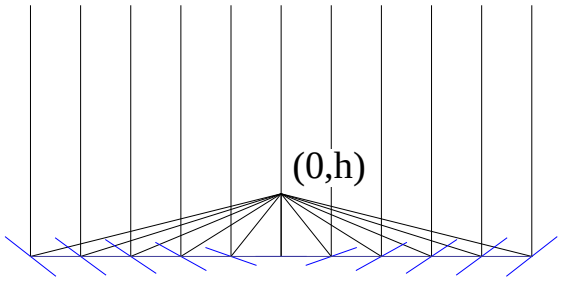
The Law of Reflection

1. First, we need to come up with a formula for the slope $R(m)$ of a vertical sunbeam reflected off of a flat mirror at slope m. The idea is to first write m in terms of the angle θ between the mirror and the vertical sunbeam, then to solve for the angle ψ that the reflected sunbeam makes with the horizontal, then to write the slope of the reflected sunbeam in terms of ψ. This diagram should help:



You'll need to use a trig identity (or two) to write the answer in terms of the original slope m.

2. A curved mirror reflects a sunbeam striking the mirror at $(x, f(x))$ as if the mirror was a flat mirror with slope given by the slope $f'(x)$ of the tangent line to $f(x)$. If we want the reflected ray to strike the collector at $(0, h)$, the slope of the line joining $(x, f(x))$ and $(0, h)$ must be the same as the slope $R(f'(x))$ of the reflected ray. Use this fact to write down an equation involving $f(x)$, $f'(x)$, and h.
3. Verify that $f(x) = cx^2$ solves the equation for $f(x)$ and $f'(x)$ that you got in the last problem for some value of c. What c works?
4. Draw a real-world conclusion from your work above: your client has a solar collector mounted 1.5 meters above the ground. What mirror should the client use to focus sunlight on this collector?
5. (Bonus Credit: Fresnel Mirrors) Large curved mirrors are heavy and expensive, since so much material must be precisely cast and supported in the air. A cheaper and lighter alternative is to mount a collection of small flat mirrors at ground level, as in the picture below, with varying slopes which focus sunbeams at the collector location of $(0, h)$. This kind of setup is called a *Fresnel Mirror*.



All the mirrors are mounted at ground level ($y = 0$). Find a formula for the slope of a mirror at $(x, 0)$ which reflects a vertical sunbeam to the collector at $(0, h)$.