# Math 2250 Lab #2 : Throwing a ball

## 1. Introduction to the lab program.

Here are some general notes and ideas which will help you with the lab. The purpose of the lab program is to expose you to problems which are more complicated and open-ended than traditional calculus exercises. They will require you to really understand some applications of calculus, in particular learning much more about projectile motion than you would learn in a standard calculus class. We are doing this because learning to solve these kinds of problems is excellent preparation for the science and engineering coursework you have ahead of you and for the career you'll have once you leave college. The students who have take the labs before you asked me to add some general insights about the labs for you to read before you start. These will make even more sense once you've done the lab, but they are good pointers to get ready for the work ahead.

- Don't give up too easily (and don't be afraid of complexity)!

  Every problem that someone will pay you to solve (regardless of your major or career path) will be complicated, messy, and hard to define. All of the easy problems are now done by computers and robots. So now is the time to start learning to deal with complicated problems. This is very different from learning to do well on standardized tests of "mathematical knowledge" or "mathematical ability". The point of the lab is not to make you better at filling in the correct circles on scantron sheets– nobody hires professional circle-fillers!– it's to start the process of making you employable.

- Read the instructions (more than once)!

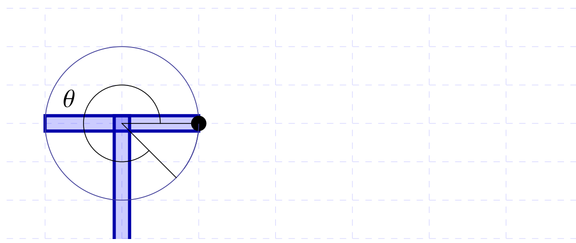
  The labs are a different kind of text than you may have seen before. You may have to read the instructions several times before you understand the question. This is something that you should expect. It often helps to read the instructions aloud.

- Talk with your classmates (even if none of you know the answer)!

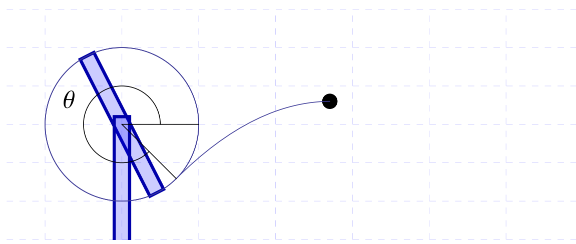
  It is easy to think about talking as a process where knowledge is transmitted from someone who has it to someone who doesn't. This is often true. But talking can also be a way for a group of people to cooperatively generate new knowledge that nobody in the conversation had when the conversation started. In these conversations, you don't have to be the most advanced student in the group to make an important contribution– sometimes asking the right question can be more important than having all the answers. Learning how to have these kinds of conversations is one of the most important things that you can learn from this class.

## 2. Client Requirements

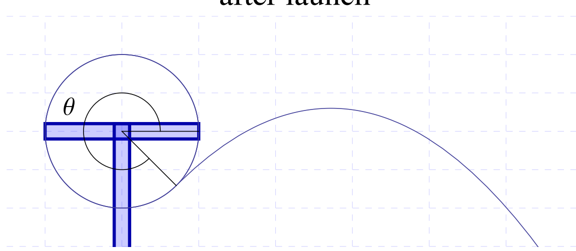
Our client is designing a robot to throw a ball across the table into a cup. The robot will have a rotating arm of length $r\,\mathrm{cm}$ mounted on a stand at height $h\,\mathrm{cm}$ above the table surface. The arm will spin at $k\,\mathrm{rpm}$ and release the ball at angle $\theta_0$ (radians).The pictures below show the basic setup:




The ball at start



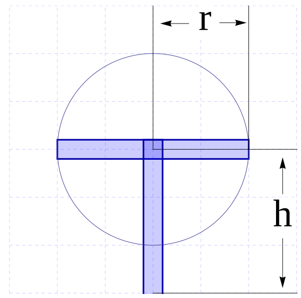

after launch




at landing.

Our assignment is to explain to the client how to deduce the landing position of the ball in terms of $r$, $h$, $k$, and $\theta_0$, keeping track of units along the way, and explaining to the client the units of the answer.

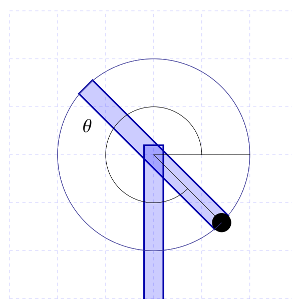
1. If the arm is rotating at $k$ rpm and the angular position of the arm is $0$ radians at time $0$, what is the *angular* position $\theta$ of the arm (in radians) at time $t$?
2. Use this answer to determine the $x$ and $y$ coordinates of the end of the arm at time $t$, and to write these as functions $\mathrm{px}(t)$ and $\mathrm{py}(t)$ giving the position of the arm as a function of time. Be sure to specify where the axes of your $x$-$y$ coordinate system are located, the (distance) units of $\mathrm{px}(t)$ and $\mathrm{py}(t)$, and the (time) units of $t$. The diagram at the top of the next page should help:




You'll need to use some trig to locate the $x$-$y$ coordinates of the end of the arm at time $t$ in terms of the angular position of the arm.

3. Use the position functions $\mathrm{px}(t)$ and $\mathrm{py}(t)$ to find *velocity* functions $\mathrm{vx}(t)$ and $\mathrm{vy}(t)$ for the ball while it is on the arm. Be sure to specify the units for the functions $\mathrm{vx}(t)$ and $\mathrm{vy}(t)$.

4. Recall that the ball will be released from the arm when the arm reaches angle $\theta_0$ (the "release angle"). In the picture below, we show $\theta_0 = 7\pi/4$, but this is just an example– we need to leave $\theta_0$ as a variable for now.

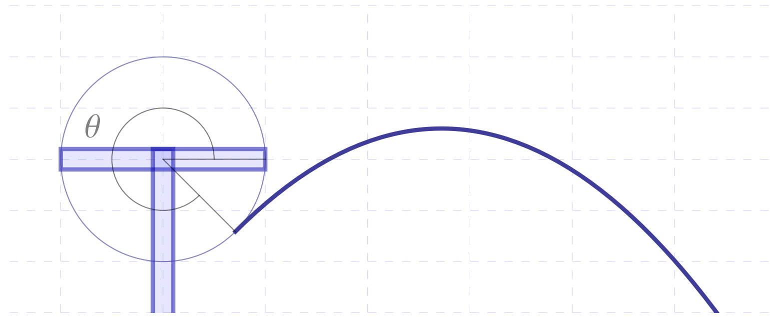



First, solve for the *time* of release $t_0$ in terms of the angle of release $\theta_0$ and the rotational speed of the arm $k$ (the answer to problem 1 will help). Now use your position and velocity formulas from the last problem to find the $x$ and $y$ position $(\mathrm{PX}_0, \mathrm{PY}_0)$ and $x$ and $y$ velocity $(\mathrm{VX}_0, \mathrm{VY}_0)$ of the ball at the moment it is released from the rotating arm.

5. We are now going to use $(\mathrm{PX}_0, \mathrm{PY}_0)$ and $(\mathrm{VX}_0, \mathrm{VY}_0)$ as the initial conditions for a new *projectile* motion problem which describes the flight of the ball after it leaves the arm. Let's reset our time coordinate so we assume that the ball leaves the arm at time $t = 0$. From now on, the time coordinate $t$ will measure the time elapsed since the ball was released.

   Write down new position functions $\mathrm{PX}(t)$, $\mathrm{PY}(t)$ which describe the ball in flight in terms of the position $(\mathrm{PX}_0, \mathrm{PY}_0)$ and velocity $(\mathrm{VX}_0, \mathrm{VY}_0)$ of the ball at the instant it is released from the arm. Use $g$ for the gravitational constant, since you will later want to use a numerical value for $g$ which corresponds to your distance and time units.

   Even though you figured out formulas for $(\mathrm{PX}_0, \mathrm{VX}_0)$ and $(\mathrm{VX}_0, \mathrm{VY}_0)$ in terms of $r$, $h$, $\theta_0$, and $k$ in the last problem, it's probably better not to substitute these formulas in yet. Your new equations $\mathrm{PX}(t)$ and $\mathrm{PY}(t)$ describe the parabolic flight of the ball after it leaves the arm (shaded heavily in the picture at the top of the next page):




6. Use your $\mathrm{PY}(t)$ function to determine the length of time it takes for the ball to hit the ground after it is released from the arm (in terms of $\mathrm{PX}_0$, $\mathrm{PY}_0$, $\mathrm{VX}_0$, $\mathrm{VY}_0$, and $g$). Note that you're solving a quadratic here, so include some discussion about *which* of the two solutions is the correct one.
7. Use your $\mathrm{PX}(t)$ function to determine the $x$ position of the ball at this point in time (again, in terms of $\mathrm{PX}$, $\mathrm{PY}$, $\mathrm{VX}$, $\mathrm{VY}$, and $g$). This is the landing position function.
8. Now give a recap for the client, explaining how to take input values of $r$ (in $\mathrm{cm}$), $h$ (in $\mathrm{cm}$), $k$ (in $\mathrm{rpm}$) and $\theta_0$ (in radians) and calculate a landing position for the ball. Be sure to tell the client the units of the landing position, and the coordinate system in which the landing position is specified. Give $x$ and $y$ coordinates for the landing position, the base of the robot, and the center of the arm. Illustrate your method by computing the specific landing location for the parameters $h = 15\,\mathrm{cm}$, $r = 11\,\mathrm{cm}$, $k = 150\,\mathrm{rpm}$, and $\theta = 7\pi/4$.
9. (Bonus Credit) Use *Mathematica* or *Wolfram Alpha* to plot the distance function as a function of the launch angle $\theta$ for several realistic values of $k$, $r$, and $h$. Design a realistic tabletop robot which can throw a ball 1 meter: what (approximate) values of $r$, $k$, and $h$ will do? What is the (approximate) launch angle $\theta$ for your design?

## 3. Grading Standards for This Lab

Basically, you will be graded on your explanation to the client of your procedures for solving this problem. The example computation is not the point of the lab, but it is the final test of your methods. If your methods (as carried out on the example) fail to work, it will cost you one letter grade.

A. The lab is clearly explained and easy to follow for any client. The procedures are "ready to code". Explanations show a deep conceptual understanding of the mathematics of the various problems. Algebra and computations are clear correct and the equation found for landing position is clear and well-supported and results in a landing location within 2.0 cm of the correct location in the example computation.

B. EITHER The lab is explained, but the explanation requires the client to be familiar with the problem or requires significant additional work in order for the client to implement the solution. The example computation is supported, but may be hard to follow. The landing location for the example is within 2.0 cm of the correct value OR The explanation of the lab meets the standards for the 'A' grade, but the example landing location is not within 2.0 cm of the correct location.

C. EITHER The explanation in the lab is incomplete and/or does not show a conceptual understanding of the problem. Procedures are incompletely specified or require substantial effort to interpret. However, the example computation is supported and results in a landing location for the example within 2.0 cm of the correct value. OR The explanation meets the standards for the 'B' grade, but the example landing location is not within 2.0 cm of the correct location.

D. Grades of 'D' or below are assigned to labs where the exposition is exceptionally poor and/or the computed landing location of the example ball is not near the correct location.

## 4. Group work Standards for this Lab

The rule for group work on the labs is simple:

Work together, write separately, acknowledge everything.

This means that you need to write up everything in your lab yourself *including calculations*. Of course, it is fine if your calculations reproduce the results of other members of your lab group– you are welcome to share approaches and ideas with the other students, and correct calculations following the same approach *should* lead you to the same numbers. However, **you may not share electronic files of any kind.** In particular, you may not share a Google doc or Word file containing a draft of your lab, Mathematica or MATLAB code, or screenshots. You may make individual handwritten notes about your calculations as a group, but typing together is strongly discouraged; you may mean well, but in practice it's very hard to avoid copying each other's work.

Similarly, it's encouraged for other students to teach you how to use *Mathematica*, but you need to have the experience of facing down the computer yourself– I don't want you to take a screenshot of somebody else's computer work and turn it in as your own.

You are welcome to complete the lab using alternative technology (such as your graphing calculator), but I encourage you to learn Mathematica or Wolfram Alpha. At this point in your college career, you're nearing the limits of the graphing calculator's capabilities, and it's time to move up.

You may turn in work prepared on the computer (using Word or something similar), handwritten work, or any mixture of the two. It's sometimes convenient to mix the two by taking photographs of your handwritten work and including them in the lab. (Of course, you can't take photographs of someone else's handwritten work and include them in the lab!)

## 5. How to turn in the lab

In order to keep things organized, there are a couple of procedures to follow when turning in the lab:

- You must print and turn in a printout of your lab work. Please proofread the printout; it can require some debugging to get symbols and formulas to print correctly from MS word. It's ok to write in by hand anything that's missing.

- You must **also** turn in an electronic copy of the lab. The electronic copy should be a PDF document with a filename in the form `lastname-lab-1-F2017.pdf`.
- You can email the electronic copies to me.

## 6. Acknowledgements

Many thanks to the Fall 2013 class for helping me refine this lab assignment. In particular, Spenser Daughdrill caught some important typos, and Dionne Aikhionbare helped me realize that I hadn't specified units for the inputs, while Salem Rustom made me be a lot more clear about the type of writeup I was looking for.