# Math 4600 - Probability

Welcome. ~~Names~~

Syllabus.

Website.

Google Calendar.

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~~Randomness is~~

Definition. When something happens at <u>random</u> there are ~~a~~ several potential <u>outcomes</u>. Exactly one outcome occurs. An <u>event</u> is a collection of outcomes.

Example. You flip a coin.

Outcomes: {heads, tails}

Events: $\emptyset$, {heads}, {tails}, {heads, tails}

Definition. $\emptyset$ is the empty set (no outcome)
S = all outcomes is called the
<u>sample space</u>.

Lemma. $\emptyset$ never happens. S always happens.

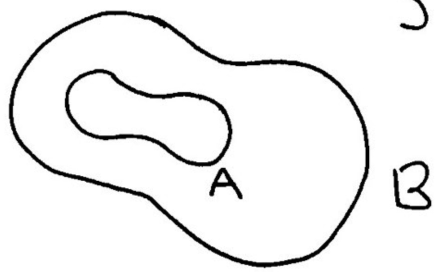
Example. You roll a 6 sided die.

Sample space: (students)

Events: the number is even
the number is 4
the number is not 4

Definition. Event $A$ is a <u>subset</u> of event $B$, written $A \subset B$, if every outcome in $A$ is also an outcome in $B$.

S (all outcomes)




~~Example A mother has delivers a baby~~

A vending machine in Boyd sells soda and water.

a) You purchase ~~a~~ 1 drink at random.

b) You purchase ~~some~~ $\geq 1$ drinks at random.

~~Outcomes:~~ Sample space:

a) $\{(s),(w)\}$

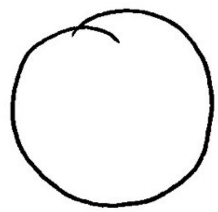
b) $\{(s),(w),(s,s),(s,w),(w,s),(w,w),(s,s,s),\ldots\}$

Example. You pick a house in Athens at random and count the number of cats in the house.

Sample space: (students)

Event that there are at most 5 cats: (students'

Example. You throw a dart at a circular dartboard and note where it lands.

Sample space: 

To describe this, we use set notation

$$\{ \text{(things)} \mid \text{(conditions on things)} \}$$

Example. The unit disk is

$$\{(x,y) \in \mathbb{R} \mid x^2+y^2 \leq 1\}$$

Definition. The union of events A and B

$$\{s \in S \mid s \in A \ \underline{\text{or}}\ s \in B\} = A \cup B$$

The intersection of events A and B

$$\{s \in S \mid s \in A \ \underline{\text{and}}\ s \in B\} = A \cap B$$

Example. You shuffle a deck of cards and draw cards until you find the ace of spades. without replacement

"without replacement" means that you don't put cards back into the deck after you draw them.

Event: three cards are drawn before you find the ace.

$$\{ (x_1, x_2, x_3) \mid \text{the } x_i \text{ are all different and } x_3 = A\spadesuit \}$$

Sample space:

$$S = \{ (A\spadesuit) \} \cup \{ (x_1, x_2) \mid x_i \text{ distinct}, x_2 = A\spadesuit \}$$
$$\cup \{ (x_1, x_2, x_3) \mid x_i \text{ distinct}, x_3 = A\spadesuit \}$$
$$\vdots$$
$$\cup \{ (x_1, \ldots, x_{52}) \mid x_i \text{ distinct}, x_{52} = A\spadesuit \}$$

This space is large, but finite.

Example. You shuffle a deck of cards and draw until you find A♠, replacing the card after each draw

Event: exactly 3 cards are drawn before you find the ace

$$\{ (x_1, x_2, x_3) \mid \underline{\text{only}}\ x_3 = A♠ \}$$

Sample space: We say $B_K$ is the event "A♠ appears on the Kth draw"

$$B_K = \{ (x_1, \dots, x_K) \mid \underline{\text{only}}\ x_K = A♠ \}$$

Then

$$S = \left( \bigcup_{K=1}^{\infty} B_K \right) \cup \text{A♠ is never drawn}$$

We now introduce more notation

Definition. For any event A, the complement $A^c$ is the set of all outcomes in S which are not in A.

$$A^c = \{ s \in S \mid s \notin A \}$$

We also call this event

"not A", "$\neg A$", or $S \setminus A$

where $\setminus$ is "set minus".

We can now relate unions, intersections and complements with two beautiful theorems!

Theorem. (De Morgan's First Law)

For a finite or infinite collection of events $A_1, \ldots$

$$\left( \bigcup_j A_j \right)^c = \bigcap_j A_j^c$$

Theorem. (De Morgan's Second Law)

$$\left( \bigcap_j A_j \right)^c = \bigcup_j A_j^c$$

Example. A card is drawn from a deck

$A_1$ = the card is a heart

$A_2$ = the card is a face card

then

$(A_1 \cup A_2)^c$ = the card is not (a heart <u>or</u> a face card)

$A_1^c \cap A_2^c$ = the card is (not a heart)
<u>and</u> (not a face card).

Students: Illustrate the second law.