Math 4500/6500 Syllabus

Dr. Jason Cantarella
Office: Boyd 448
Office phone: 542-2610
jason@math.uga.edu

Our classroom: Boyd 222
11:15-12:05 MWF
http://www.jasoncantarella.com
"Courses", "Math 4500"

Office hours: M 2-4

Book: Ward Cheney and David Kincaid, *Numerical Mathematics and Computing* (6th edition).

Part I: Numbers and Roundoff Error

- Roundoff error. Taylor series review. (1 week)
  Initial examples. Standard forms of Taylor's theorem. O and o notation.
- Floating point representation. (1 week)
  Definition. Machine numbers. Machine epsilon. Loss of precision.

Part II: Rootfinding

- One-dimensional rootfinding without derivatives. (1 week)
  Bisection. False position method. Brent's method.
- Rootfinding with one derivative. (1 week)
  Newton-Raphson method. Convergence and stability. Systems of equations.
- Special cases. (1 week)
  Rootfinding for polynomials. Deflation. Horner's algorithm. Fixed point method.

Part III: Interpolation and Numerical Differentiation

- Polynomial Interpolation. (1 week)
  Lagrange and Newton polynomials. Calculating coefficients. Vandermonde Matrix. Chebyshev polynomial. Curve fitting.
- Spline Interpolation. (1 week)
  Cubic splines. Bezier curves. NURBS. Splines and animation.
- Interpolation Error. (1 week)
  Runge function. Choice of nodes. Error bounds.
- Numerical Differentiation. (1 week).
  Taylor's theorem estimates. Richardson Extrapolation. Numerical estimates of curvature and torsion for space curves.

Part IV: Numerical Integration

- Review of Calculus. (1 day)
  Upper and lower sums. Trapezoid rule.
- Recursive trapezoid rule and Romberg Algorithm. (1 week)
  Subdivision. Error estimates. Euler-Maclaurin Formula.
- Simpson's Rule and Adaptive Simpson's Rule. (2 days)
  Derivation of Simpson's rule. Subdivision. Newton-Cotes formulae.
- Gaussian quadrature. (1 week)
  Placement of Gauss nodes. Legendre polynomials. Integrating up to a singularity. Adams-Bashforth-Moulton formulae.
- Very high precision numerical integration. (1 week)
  Methods. PSLQ. Formula for digits of pi. Inverse symbolic calculator.
- Numerical Integration in Multiple Dimensions. (1 week)
  Quadrature rules. TRIEX. Geometric examples.

0.1. **Prerequisites.** Students are expected to have MATH 2500 and MATH 3000, as well as (informally) CSCI 1301 or equivalent programming experience. Numerical mathematics is done on computers, so programming will be a substantial part of the course.

0.2. **Course Goals.** Students will develop fluency with fundamental techniques in numerical mathematics and apply them to some real-world problems. The most important outcome of the course is for students to understand the role and analysis of numerical error in computations.

0.3. **Disclaimer.** The course syllabus is a general plan for the course; deviations announced to the class by the instructor may be necessary.

0.4. **Principal Course Assignments and Mathematica.** Homework will be due more or less weekly. Much of the homework will use Mathematica. The Math Department has a site license for Mathematica 7 which allows students to get copies for personal or home use. This course assumes that you have access to a computer on which you can install Mathematica and use it for some longer projects and assignments. The course will end with a final project (considered a take-home final exam) which will involve significant Mathematica programming.

0.5. **How to obtain Mathematica.** To request a home use license please visit `http://www.wolfram.com/siteinfo/homeuse/` This is a two part process.

(1) Enter the main license number first L3269-0534 Choose the radio button "A New home-use license", then continue. Next enter your contact information. Scroll to the bottom of the page and click submit.
(2) Send an email to `sohayl@uga.edu` indicating that you are from the Math department and that you have requested a home use license. Once Sohayl approves you, you will get the license code by email.

Once you have your license, you can download your copy of Mathematica by visiting `https://sitesoft.uga.edu/mathematica` with your MyID and password.

0.6. **Grading and WP/WF policies.** We plan to have two take-home exams: at the conclusion of parts II and III. The overall course grade is computed from homework, exam, and final grades by the formula:

(1) 20% for each regular exam.
(2) 20% for the final project (which counts as a take-home final exam).
(3) 40% for homework assignments.

In order to receive a grade of "WP" before the first exam, you must have scored at least 50 % of the homework points available by the date of withdrawal. After the first exam, this policy will remain in force for a two week grace period. After this period expires, you must have scored at least 50 % of the homework points **and** 50 % of the first exam points in order to receive a grade of "WP".

0.7. **Attendance Policy.** Students are expected to attend class. Students who miss more than 6 classes (two weeks of class) may be withdrawn from the course by the instructor. The WP/WF policy above applies to students withdrawn by the instructor.

0.8. **Academic Honesty.** As a University of Georgia student, you have agreed to abide by the University's academic honesty policy, "A Culture of Honesty," and the Student Honor Code. All academic work must meet the standards described in "A Culture of Honesty" found at: `www.uga.edu/honesty`. Lack of knowledge of the academic honesty policy is not a reasonable explanation for a violation. Questions related to course assignments and the academic honesty policy should be directed to the instructor.

It is perfectly acceptable to work on homework problems in groups in this course. However, the help you should get from your fellow students should enable you to complete the problem on your own. Recruiting another student to complete the homework for you, or to simply provide answers to the problems, is a violation of the honesty policy.

0.9. **Required Course Material.** The course textbook (Cheney and Kincaid, *Numerical Mathematics and Computing*, 6th edition) is required for the course. In addition, it is assumed that each student owns a computer on which they can install Mathematica.

0.10. **Make-up Examinations.** This course has only take-home examinations. These are due in class. Late examinations will not be accepted without a very good reason (usually medical or legal). **Makeup examinations will not be given.**