The Patriot Missile failure.

In 1991, during the first Gulf war with Iraq, a missile defense system was deployed to protect US Army forces in Saudi Arabia. The threat was a Soviet made SCUD ballistic missile.

The missile defense system, called Patriot, used radar to track incoming SCUDs and fired a missile to intercept in midair.

The Patriot's computer kept time in increments of 0.1 seconds in an integer counter.

However, it multiplied this count by a 24 bit fixed point representation of 0.1 to convert time to a floating point representation.

Converting fractions to decimals.

Recall that

$$a_0 + a_0 r + a_0 r^2 + \cdots = \frac{a_0}{1-r}$$

if $|r| < 1$.

We can use this sum to work out repeating representations of fractions.

Example.

$$3.\overline{3} = 3 + 3\times10^{-1} + 3\times10^{-2} + \dots$$

$$= \frac{3}{1-\frac{1}{10}} = \frac{3}{\frac{9}{10}} = \frac{10}{3}.$$

$$\frac{1}{11} = \frac{9/100}{1 - 1/100}$$

$$= \frac{9}{100} + \frac{9}{100}\times10^{-2} + \frac{9}{100}\times10^{-4} + \dots$$

$$= .09 + 0.0009 + 0.000009 + \dots$$

$$= .09090909\dots$$

In base 2 floating point, ④

$$\frac{1}{10} = \frac{1}{2} \times \left(\frac{3/16}{1 - 1/16} \right) = \cancel{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{3 \times 2^{-4}}{1 - 2^{-4}} \right)$$

$$= \frac{1}{2}\left(3 \times 2^{-4} + 3 \times 2^{-8} + 3 \times 2^{-12} + \ldots \right)$$

$$= \cancel{\frac{1}{2}(0.0011}$$

$$= \frac{1}{2}\left(0011_2 \times 2^{-4} + 0011_2 \times 2^{-8} + \ldots \right)$$

$$= 0.0\overline{0011}$$

$$= \cancel{1.1001100 \times 2^{-4}}$$

Expressed as a ~~single~~ 24 bit fixed ~~precision~~ number, this was chopped to

~~0.0 0011 0011 0011 0011 0011~~

$0.000\overbrace{1100}^{4}\overbrace{1100}^{8}1100\,1100\,1100$

which gave a truncation error of

$$0.\overline{1100} \times 2^{-24} = \frac{8}{10} \times 2^{-23}$$

On February 25, 1991 the Patriot battery in Dharan, ~~Saudi~~ had been operating for ~~m~~ about 100 hours, building up a truncation error of

$$3.6 \times 10^{6} \times \frac{8}{10} \times 2^{-23} = 0.34\,s.$$

The Patriot had software modifications which calculated

the time correctly using more precision. However they were not used everywhere they should have been, leading to the ~~comparison~~ subtraction of a correct and truncated time, for two radar pulses.

An incoming SCUD, moving at ~1676 m/s, was incorrectly predicted to be ~500 m from its actual position. The Patriot failed to lock on, and the SCUD hit a U.S. Army barracks, killing 28 and injuring ≥100 servicemembers.