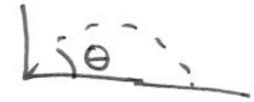
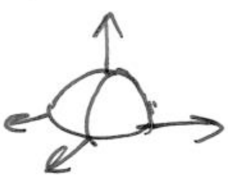
Math 2500 - Lecture 1.

About me, syllabus, name game/pics/anki 25 min

Why multivariable calculus? 25 min

Derivatives. Maximize $f(\theta) = \cos\theta \sin\theta$. 
(car launch problem)
Take derivative.
Find critical point.
What this means for $f(\theta,\psi) = \cos \theta^2 \psi$.

Integrals. Integrate $\int_{-1}^{1} 1 - x^2 \, dx$. 
area under line
Integrate $\int_{x^2+y^2 \leq 1} 1-(x^2+y^2)\,dx\,dy$ 
volume under surface

Later in course we'll have some entirely new ideas.

Real-world problems. Lunocet. Knot-tightening.

Math 2500 - Lecture 2 - Vectors.

Definition. A <u>point</u> in $\mathbb{R}^n$ is an ordered list of n numbers $(x_1, \dots, x_n)$.

Examples. $\mathbb{R}^2$, $\mathbb{R}^3$, $\mathbb{R}^{467}$

Definition. A <u>vector</u> $\vec{v}$ in $\mathbb{R}^n$ is a directed line segment joining two points in $\mathbb{R}^n$.

Convention. If the first point is not given, we assume it's <u>the origin</u>.

Examples. $\vec{v} = \overrightarrow{(2,4)\ (3,5)}$, $\vec{v} = (7,8)$, $\vec{v} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$.

Definition. The <u>length (or magnitude)</u> of $(x_1, \dots, x_n) = \vec{x}$ is $\sqrt{x_1^2 + \dots + x_n^2}$, denoted $\|\vec{x}\|$.

Definition. $\vec{u} + \vec{v} = (u_1 + v_1, \dots, u_n + v_n)$, $k\vec{u} = (ku_1, \dots, ku_n)$.

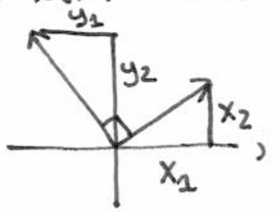
Note: We call a number k a <u>scalar</u> to avoid confusion.

Example. Addition, with pictures, scalar mult, with pics.

Example. Distance formula, midpoint, medians of triangle.

Exercises. $\vec{x} + \vec{y} = \vec{y} + \vec{x}$. $(\vec{x} + \vec{y}) + \vec{z} = \vec{x} + (\vec{y} + \vec{z})$. $c(\vec{x} + \vec{y}) = c\vec{x} + c\vec{y}$.

Math 2500 - Lecture 3 - Dot product, cross product



, by similar triangles $\frac{x_2}{x_1} = \frac{-y_1}{y_2}$, or $x_1y_1 + x_2y_2 = 0$.

Definition. The <u>dot product</u> $\vec{u}\cdot\vec{v} = u_1v_1 + \ldots + u_nv_n$.

Theorem. The angle θ between $\vec{u}$ and $\vec{v}$ is given by

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}.$$

Proof. Law of cosines. 

, $c^2 = a^2 + b^2 - 2ab\cos\theta$.

Corollary. $\vec{u} \perp \vec{v} \iff \vec{u}\cdot\vec{v} = 0$.

Examples. $(3,2)$, $(4,6)$, $(3,2)$, $(7,13)$

Exercises. $\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$, $(c\vec{u})\cdot\vec{v} = \vec{u}\cdot(c\vec{v}) = c(\vec{u}\cdot\vec{v})$, $\vec{u}\cdot(\vec{v}+\vec{w}) = \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w}$,
$\vec{u}\cdot\vec{u} = \|\vec{u}\|^2$, $\vec{0}\cdot\vec{u} = \vec{0}$.

Proposition. $\text{proj}_{\vec{v}}\,\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\,\vec{v}$.

Examples. $|\vec{x}\cdot\vec{y}| \leq \|\vec{x}\|\,\|\vec{y}\|$ (Cauchy-Schwartz)

Definition. The <u>cross product</u> of two vectors $\vec{u}, \vec{v} \in \mathbb{R}^3$ is

$$\vec{u}\times\vec{v} = (x_2y_3 - x_3y_2,\ x_3y_1 - x_1y_3,\ x_1y_2 - x_2y_1).$$

Right-hand rule. $\vec{u}\times\vec{v} = (\|\vec{u}\|\,\|\vec{v}\|\sin\theta)\,\vec{n}$.

Corollary. $\vec{u}$ and $\vec{v}$ are parallel $\iff \vec{u}\times\vec{v} = \vec{0}$.

Exercises. $(r\vec{u})\times(s\vec{v}) = rs(\vec{u}\times\vec{v})$, $\vec{u}\times(\vec{v}+\vec{w}) = \vec{u}\times\vec{v} + \vec{u}\times\vec{w}$.
$\vec{u}\times\vec{v} = -\vec{v}\times\vec{u}$, $\vec{0}\times\vec{u} = \vec{0}$.

Examples. $(1,3,1)\times(2,4,5)$, $(1,3,1)\times(2,6,2)$

Proposition. $\vec{u}\times\vec{v}$ is the area of a parallelogram.

Definition. $(\vec{u}\times\vec{v})\cdot\vec{w}$ is called the triple scalar product.

Proposition. $(\vec{u}\times\vec{v})\cdot\vec{w}$ = volume of box.