

# MATH 2260

Midterm Exam III

November 18, 2008

**NAME (please print legibly):** \_\_\_\_\_

**Your University ID Number:** \_\_\_\_\_

Please complete all questions in the space provided. Draw a box around your final answer. You may use the backs of the pages for extra space, or ask me for more paper if needed. Work carefully, and neatly (part of your grade will be based on how well your work is presented).

Try to complete the problems you find easier before going back to the harder ones. Good luck!

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	15	
6	10	
<b>TOTAL</b>	<b>65</b>	

**1. (10 points)** Find the limit of the **sequence**

$$\lim_{n \rightarrow \infty} \frac{\ln n}{\ln 2n}$$

**2. (10 points)** Suppose that  $x > 1$ . Does the **series**

$$\sum_{n=0}^{\infty} \frac{1}{x^n}$$

converge? If so, find the sum of the series as a function of  $x$ .

**3. (10 points)** Does the series

$$\sum \frac{5^n}{3^n + 2}$$

converge or diverge?

**4. (10 points)** Remember that

$$\sin^2 x = \frac{1 - \cos 2x}{2}.$$

Use this to find the Taylor series for  $\sin^2 x$  at  $a = 0$ .

**5. (15 points)** You (hopefully!) know the Taylor series for  $\cos x$  at  $a = 0$ . Take a moment to write it down. Pause to appreciate your work. Now forget about it and answer the first two parts of this three part question. (You can remember it when you get to part 3).

- Find the Taylor series for  $\cos x$  at  $a = \pi/4 \simeq 0.785$ .
- Find the Taylor approximation  $P_3(x)$  for  $\cos x$  with  $a = \pi/4$ .
- Use the 3rd order Taylor polynomial  $P_3(x)$  with  $a = \pi/4$  and  $P_3(x)$  with  $a = 0$  to estimate  $\cos 1$ . Use your calculator to check the results. Which is better? Why?

**6. (10 points)** Use Taylor series to find a polynomial approximation for

$$f(x) = \int_0^x \arctan t \, dt.$$

which has error less than  $10^{-2}$  on  $[0, 1]$ . Hint: You can estimate the error of a Taylor polynomial derived from an **alternating** Taylor series without using the Taylor Remainder Theorem. (And it's probably a good idea in this case.)