MATH 4600

Final Exam I

May 2, 2019

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Please complete all questions in the space provided. You may use the backs of the pages for extra space, or ask me for more paper if needed. This exam will be graded on:

- Correctness of computations.
- Clarity of explanation of procedure.
- Correctness of procedure.

A correct answer obtained using an incorrect or poorly explained procedure will not be graded for full credit. Please feel free to write as much as you like. Work carefully, and try to complete the problems you find easier before going back to the harder ones. Good luck!

QUESTION	VALUE	SCORE
1	10	
2	15	
3	10	
4	20	
5	10	
6	10	
7	15	
TOTAL	90	

1. (10 points) The random variable X is the indicator variable for the event "It rains", and the random variable Y is the indicator variable for the event "It's complicated". The joint pmf of X and Y is

$p_{X,Y}(x,y) =$

	X=0	X=1
Y=0	0.1	0.3
Y=1	0.2	0.4

I am only happy when it rains *and* it's complicated. Suppose that it rains. Find the probability that I'm happy. (5pts)

ANSWER: ____________________

Now suppose that it's complicated. Find the probability that I'm happy. (5pts)

ANSWER: ____________________

2. (15 points) During the month of January in Athens, GA, the average daily rainfall is 0.12 inches. If there are more than 3 inches of rain in any one day, my basement will flood. Use Markov's inequality to find an upper bound on the probability that my basement will flood on January 15 of next year. (5pts)

ANSWER: ____________________

During the month of January in Athens, GA the average *monthly* rainfall is 4.05 inches. During January 1996, the monthly rainfall in Athens was 6.71 inches. Use Markov's inequality to find an upper bound on the probability of a total monthly rainfall ≥ 6.71 inches in January. (5pts)

ANSWER: ____________________

A friend claims that in January 1996, CIA cloud seeding experiments caused unusual rainfall. How unusual was the rainfall in January 1996? Does your computation above provide weak or strong evidence for your friend's story? (Note: There is no right answer here; this is graded on your ability to reason from evidence.) (5pts)

3. (10 points) A storm of a certain intensity A is estimated to occur once in every 500 years in Houston, TX. The number of such storms in a 10 year interval is a Poisson random variable X. Find the pmf $p_X(x)$. (5pts)

ANSWER: ____________________

Find the probability that 3 or more such storms strike Houston in a 10 year interval. (5pts)

ANSWER: ____________________

4. (20 points) Suppose that each student in a class of N students has a birthday which is equally likely to fall on each of 365 days. Let X_{ij} be the indicator variable for the event "student i and student j have the same birthday" (these are defined only for $i < j$, so that each pair of students has a unique indicator variable).

Find the expectation $E(X_{ij})$. (5pts)

ANSWER: ______________________

Let X be the number of pairs of students in a class of N students with the same birthday.
Find $E(X)$. (5pts)

ANSWER: ______________________

Are the variables X_{ij} *pairwise* dependent or *pairwise* independent? (5pts)
(Note: *Joint* independence is different from *pairwise* independence.)

ANSWER: ______________________

Let X be the number of pairs of students in a class of N students with the same birthday.
Find $\mathrm{Var}(X)$. (5pts)

ANSWER: ______________________

5. (10 points) Suppose we are evaluating a medical test for a certain disease. We let D be the event that "a given person has a disease" and S be the event that "a given person tests positive for the disease".

Let C ("correctly diagnosed") be the event that a person has the disease *and* tests positive.
Let I ("incorrectly diagnosed") be the event that a person has the disease, *but* does *not* test positive.

Prove that

$$P(C) > P(I) \quad \iff \quad \frac{P(D|S^c)}{P(D|S)} < \frac{P(S)}{P(S^c)}.$$

ANSWER: ______________________

6. (10 points) A Markov model for Moody's ratings of commercial short-term bonds has two states "prime" and "non-prime". Moody's rerates the bonds at 30 day intervals. The transition probabilities are

	P	NP
P	0.7	0.3
NP	0.1	0.9

The initial population of bonds is 98% prime and 2% non-prime. The bonds are then rerated three times over the course of the next 90 days. After the third rerating is complete, what is the distribution of prime and non-prime bonds in the population? (5pts)

ANSWER: ________________________

Find the limiting distribution of "prime" and "non-prime" bonds as the number of reratings approaches infinity. (5pts)

(more space to work)

7. (15 points) In the Watkinsville animal shelter, there are 50 animals. 45 of these animals are cats. 5 of them are Flerkens[1]. An animal rescuer gathers 17 animals and packs them into a rented van for a trip to Charleston, South Carolina, where they will be adopted. Let X be the number of Flerkens in the van. (It is safe to assume that there were no Flerkens aboard when the van was originally rented.)

What type of random variable is X? (5pts)

What is the expected number of Flerkens in the van? (5pts)

[1] A Flerken is visually identical to a cat, but actually a dangerous alien species.

What is the pmf $p_X(x)$? What is the probability that there is exactly one Flerken in the van? (5pts)
(Note: The actual expression is probably too hard for your calculator, so just
set up, but do not evaluate the pmf.)

ANSWER: ______________________