MATH 4600

Midterm Exam I

January 29, 2019

NAME (please print legibly): _____

Your University ID Number: _____

Please complete all questions in the space provided. You may use the backs of the pages for extra space, or ask me for more paper if needed. This exam will be graded on:

- Correctness of computations.
- Clarity of explanation of procedure.
- Correctness of procedure.

A correct answer obtained using an incorrect or poorly explained procedure will not be graded for full credit. Please feel free to write as much as you like. Work carefully, and try to complete the problems you find easier before going back to the harder ones. Good luck!

QUESTION	VALUE	SCORE
1	15	
2	10	
3	15	
4	10	
5	20	
TOTAL	70	

1. (15 points) A fair die is rolled, producing an integer n between 1 and 6. What is the sample space S? (5pts)

ANSWER: _____

Give an example of an event A containing 3 outcomes and an event containing 2 outcomes.(5pts)

ANSWER: _____

Define *disjoint* events and state whether event A and event B are disjoint.(5pts)

ANSWER: _____

2. (10 points) State the inclusion-exclusion rule for three events, A, B, and C (5pts):

ANSWER: _____

Suppose we know that $P(A \cup B \cup C) = 0.9$, $P(B) = 0.6$, $P(C) = 0.3$, $P(A \cap B) = 0.2$, $P(A \cap C) = 0.1$, $P(B \cap C) = 0.1$, and $P(A \cap B \cap C) = 0$. Find $P(A)$. (5pts)

ANSWER: _____

3. (15 points) A random integer n is chosen between 2 and 20 (with equal probabilities). Compute the probability of the following events (5pts):

A = { n is odd }, B = { n is prime }, C = {n is even}

ANSWER: _____

State the definition of *independent* events (5pts):

ANSWER: _____

(Question 3, continued) Consider the pairs of events A, B, A, C and B, C from the previous page. Which are dependent and which are independent? Justify your answer using the definition of independence for full credit. (5pts)

ANSWER: _____

4. (10 points) A bag contains 4 marbles, 2 of which are red and 2 are black. We draw the marbles from the bag (randomly) one after the other, **without replacing them**. Consider the events:

$$A = \{\text{the first marble is red}\}, \quad B = \{\text{the second marble is red}\}$$

Compute $P(A)$ and $P(B)$. (5pts)

ANSWER: _____

Are A and B independent events? Justify your answer using the definition of independence for full credit. (5pts)

ANSWER: _____

5. (20 points) State the definition of *mutual* independence for 3 events A, B, and C (5pts).

ANSWER: _____

Two (fair) dice are rolled, producing a pair of integers x_1 and x_2 between 1 and 6. Consider the three events

$$A = \{(x_1, x_2) | x_1 + x_2 = 7\} \quad B = \{(x_1, x_2) | x_1 = 3\} \quad C = \{(x_1, x_2) | x_2 = 4\}$$

Find $P(A)$, $P(B)$, and $P(C)$. Justify your answer for full credit. (5pts)

ANSWER: _____

(Continued). Are the pairs of events A, B, A, C and B, C independent or dependent? Justify your answer using the definition of independence for full credit. (5pts)

ANSWER: _____

Are all three events A, B, and C jointly (or mutually) independent? Justify your answer for full credit using the definition of mutual independence you gave above. (5pts)

ANSWER: _____