MATH 2250

Midterm Exam II

November 1, 2016

NAME (please print legibly): ____________________

Your University ID Number: ____________________

Please complete all questions in the space provided. You may use the backs of the pages for extra space, or ask me for more paper if needed. This exam will be graded on:

- Correctness of computations.
- Clarity of explanation of procedure.
- Correctness of procedure.

A correct answer obtained using an incorrect or poorly explained procedure will not be graded for full credit. Please feel free to write as much as you like. Work carefully, and try to complete the problems you find easier before going back to the harder ones. Good luck!

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
TOTAL	90	

1. (10 points) The **derivative** of $f(x)$ is given by

$$f'(x) = 6x(x+1)(x-2).$$

Find all the critical points of $f(x)$ and classify them as local mins, local maxes, or neither.

ANSWER: ____________________________

2. (10 points) Prove that $g(t) = \sin^2 t - 3t$ is always a decreasing function of t. How many solutions are there to the equation $g(t) = 5$? Why?

3. (10 points) Use L'Hospital's rule to find a formula for the limit

$$\lim_{x \to 0} \frac{\sin mx}{\sin nx}$$

in terms of m and n. Check the answer by computing some values of $\frac{\sin 5x}{\sin 3x}$ for x close to zero with your calculator. Are they close to the values your formula predicts with $m = 5$ and $n = 3$?

4. (10 points) Find the derivative of the function

$$f(x) = \frac{x}{x+1}$$

and the derivative of the function

$$g(x) = \frac{-1}{x+1}$$

What does this tell you about the two functions? Why?

5. (10 points) Suppose that the / (division) button on your calculator is broken, but $+$, $-$, and $\times$ still work, and somebody asks you to compute $1/7$. You can still find the solution using Newton's method!

Use Newton's method to approximate $1/7$ by solving the equation $f(x) = 1/x - 7 = 0$.

Write down the iteration formula, and carry out the procedure with initial guess $x = 0.1$ until you have at least 8 correct digits in your answer. (You can, of course, check the answer using the / button on your actual calculator to figure out when your 8 digits are correct.)

ANSWER: ______________________

Now write down the formula for using Newton's method to approximate $1/a$. Verify that no division is involved.

ANSWER: ____________________

6. (10 points) The building code requires that the sum of the height and width of a rectangular window be no more than 20 ft. Suppose we are trying to maximize the sum of the width of the window and the **square root** of the height of the window. What dimensions should we choose?

ANSWER: ____________________

7. (10 points) Find the indefinite integral

$$\int \sec \frac{x}{3} \tan \frac{x}{3}\, dx$$

ANSWER: ____________________________

8. (10 points) Find a function $f(x)$ with

$$f'(x) = \frac{1}{1+x^2},$$

at $f(0) = 4$.

ANSWER: ____________________

9. (10 points) Given $f(x) = ax^2 + 2bx + c$, prove that the minimum value of $f(x)$ is ≥ 0 if and only if $b^2 - ac < 0$. Use calculus.

ANSWER: __________________________

(Extra credit) Using $f(x) = (a_1 x + b_1)^2 + \dots (a_n x + b_n)^2$, in the problem above, prove that

$$(a_1 b_1 + \cdots + a_n b_n) \leq (a_1^2 + \cdots + a_n^2)(b_1^2 + \cdots + b_n^2).$$

ANSWER: ____________________________