MATH 2250

Final Exam: Part II

July 1, 2014

NAME (please print legibly): _____

Your University ID Number: _____

A correct answer obtained using an incorrect or poorly explained procedure will not be graded for full credit. Please feel free to write as much as you like. Work carefully, and try to complete the problems you find easier before going back to the harder ones. Good luck!

Remember that you are **strongly encouraged** to have a non-graphing calculator to complete the exam. Remember also that smartphone (or computer, or other device) use is **prohibited** on this exam, regardless of what you use it for.

This is Part II of the exam. Please remember to work **slowly** and **carefully** and write down each step as you proceed through the problems. When you're done this part of the exam, please check your work and hand it in. Notice that I'm still offering 3 crisp dollar bills for any unused calculators so that I can have some on hand to provide to other students.

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	20	
TOTAL	60	

1. (10 points) Consider the function

$$f(x) = \frac{2e^x}{1+x^2}$$

Find all the critical points of this function and classify them as absolute max, local max, absolute min, local min, or none of the above.

ANSWER: _____

2. (10 points) Show that the functions

$$f(x) = \frac{x}{x+1}, \quad \text{and} \quad g(x) = -\frac{1}{x+1}.$$

have the same derivative. What does that mean about these functions?

ANSWER: _____

3. (10 points) Compute the definite integral

$$\int_0^3 x^3 - 2x + 3 \, \mathrm{dx}$$

ANSWER: _____

4. (10 points) Compute the indefinite integral

$$\int \frac{4x}{\sqrt{x^2+1}} \, \mathrm{dx}$$

ANSWER: _____

5. (20 points) Fracking. Suppose we have to test N wells for contamination with an industrial chemical. One way to do to do this is to test each well separately, requiring N tests. Another method is to mix samples from x wells and test the combined sample. If the combined sample is clean, you don't have to do any more tests. If the combined sample is contaminated, each of the individual samples must be tested, for a total of $x + 1$ tests.

Using the second method, you can show using some probability theory[1] that if the probability that each well is clean is 0.99 and you have $1,000$ wells to test, the expected total number of tests is

$$y(x) = 1000\left(1 - 0.99^x + \frac{1}{x}\right)$$

Use calculus to find the group size x which minimizes the expected total number of tests, and the minimum expected number of tests. Which method is better (test each well separately, or combine samples)?

[1] which is beyond the scope of this class, alas!

(More space to work on the fracking problem)

ANSWER: _____