MATH 2260

Final Exam

May 2, 2014

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Please complete all questions in the space provided. Draw a box around your final answer. You may use the backs of the pages for extra space, or ask me for more paper if needed. Work carefully, and neatly (part of your grade will be based on how well your work is presented).

Try to complete the problems you find easier before going back to the harder ones. The questions are more or less in order of difficulty.
Good luck!

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	15	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	0	
TOTAL	125	

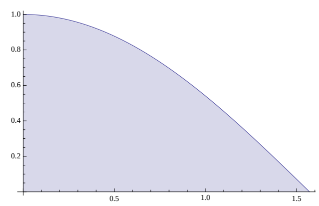
1. (10 points) Find the length of the vectors $\vec{v} = (1, 3, 2, 1)$ and $\vec{w} = (-1, 2, 5, 3)$ and the angle θ between them. (Specify whether your answer is in degrees or radians.)

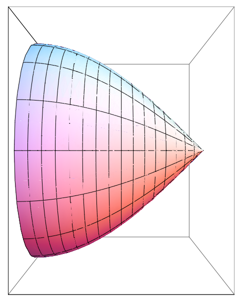
ANSWER: ____________________________

2. (10 points) Find the cross product $(1, 2, 5) \times (0, 1, 2)$.

ANSWER: ____________________________

3. (10 points) An arrowhead is manufactured from solid aluminum by revolving the portion of the curve $y = \cos x$ (cm) between $x = 0$ (cm) and $x = \pi/2$ (cm) around the x-axis, as shown below.

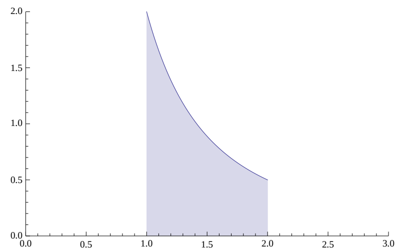




As noted above, the units are in cm. Assume that the density of aluminum is $2.7\mathrm{g}/\mathrm{cm}^3$. What is the mass (in grams) of the arrowhead?

ANSWER: ____________________________

4. (10 points) A thin plate covers the region between the x-axis and the curve $y = 2/x^2$(cm) for x between 1(cm) and 2(cm), as show in the figure below. The density of the plate is given by $\delta(x, y) = x^2$ g/cm^2.



Find the total mass of the plate in grams.

ANSWER: ______________________

Find the x coordinate of the center of mass of the plate.

ANSWER: ______________________

Bonus (+5 points). Find the y coordinate of the center of mass of the plate.

ANSWER: ______________________

5. (10 points) Find the integral

$$\int x \sin x \, \mathrm{d}x$$

ANSWER: ____________________________

Differentiate your answer above to prove that its derivative is $x \sin x$.

ANSWER: ____________________________

6. (15 points) The sinc function is defined by

$$\operatorname{sinc} x = \frac{\sin x}{x}$$

Estimate the integral

$$\int_0^2 \operatorname{sinc} x \, \mathrm{d}x$$

to within an error of 0.01 using Taylor series **or** numerical integration (your choice).

Bonus (+10 pts): Do both methods and get results which agree.

(More space to work on question #6).

ANSWER: ____________________________

7. (10 points) Does the series

$$\sum_{n=1}^{\infty} \frac{-2}{n\sqrt{n}}$$

converge or diverge? (Justify your answer with a test for full credit.)

ANSWER: ____________________________

8. (10 points) Does the **sequence** $a_n = \frac{1-n^2}{20-n^3}$ converge or diverge? If it converges, what is the limit of the **sequence**? Justify your answer for full credit.

ANSWER: ______________________

Does the **series**

$$\sum_{n=1}^{\infty} \frac{1-n^2}{20-n^3}$$

converge or diverge? (If it converges, you are **not** expected to know the limiting value of the series.) Justify your answer for full credit.

ANSWER: ______________________

9. (10 points) The first five terms for the Taylor series of $\tan x$ centered at $x = 0$ are given by

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + \cdots$$

The series converges absolutely for $x \in (-\pi/2, \pi/2)$. Use this information to give the first five terms of the Taylor series for $\sec^2 x$ centered at $x = 0$. (You are not required to simplify fractions or do the arithmetic for the coefficients.)

ANSWER: ____________________

10. (10 points) Find the Taylor series for the function

$$f(x) = \cosh x = \frac{e^x + e^{-x}}{2}.$$

centered at $x = 0$.

ANSWER: __________________________

11. (10 points) Use Taylor series to give an approximation of $\arctan 0.5$ (in radians) with error less than 0.01. (You must explain how you know the error in your approximation is small enough for full credit. "I checked it on my calculator" doesn't count.) Hint: You can rederive the Taylor series for $\arctan x$ by integrating the Taylor series for the derivative of $\arctan x$ term-by-term if you're desperate. But you should really remember the series.

ANSWER: ______________________

Compare your estimate with your calculator's result for $\arctan 0.5$. Is the difference less than 0.01? If not, try part 1 again.

ANSWER: ______________________

12. (10 points) A ship has a flat, triangular sail with corners at $(1, 4, 2)$, $(9, 7, 3)$ and $(2, 2, 20)$ (all coordinates are given in feet). What is the area of the sail in square feet?

ANSWER: ____________________________

13. (0 points) Extra Credit Problem. (10 points) Prove Oresme's theorem:

$$1 + \left(\frac{1}{2} \cdot 2\right) + \left(\frac{1}{4} \cdot 3\right) + \left(\frac{1}{8} \cdot 4\right) + \cdots + \frac{n}{2^{n-1}} + \cdots = 4.$$

Hint: Remember that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

Now differentiate both sides of the equation.