MATH 2260

Midterm Exam I

February 6, 2014

NAME (please print legibly): _____

Your University ID Number: _____

Please complete all questions in the space provided. Draw a box around your final answer. You may use the backs of the pages for extra space, or ask me for more paper if needed. Work carefully, and neatly (2 points on every problem are given for clear presentation of your work or deducted for unclear, messy, or hard-to-understand work).

Try to complete the problems you find easier before going back to the harder ones. Good luck!

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	15	
7	10	
TOTAL	75	

1. (10 points) Solve the differential equation

$$\frac{dy}{dx} = 3x^2 e^{-y}$$

2. (10 points) The triangle with vertices $(3, 0)$, $(5, 0)$ and $(5, 2)$ is revolved around the y-axis to create a solid. Find the volume of the solid.

3. (10 points) Compute the volume of a sphere of radius r using the shell method.

4. (10 points) Set up, but do not evaluate, an arclength integral for the length of the portion of the circle $x^2 + y^2 = 1$ with $y \geq 0$ (that is, the upper semicircle).

5. (10 points) The portion of the graph $y = \sqrt{2x - x^2}$ with $x \in [1/2, 3/2]$ is revolved around the x axis to create a surface of revolution. Find the area of the surface.

6. (15 points) Part 1. The Earth exerts a force of $gm(r/s)^2$ Newtons on a mass of m kilograms at a height of s meters (from the center of the Earth), where r is the radius of the Earth. Find the work required to raise a spacecraft of mass m from a height of a meters (from the center of the Earth) to a height of b meters (from the center of the Earth) in terms of g, r, m, a, and b.

ANSWER: _____

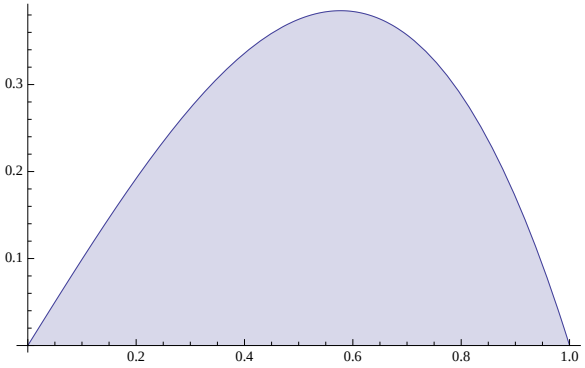
Part 2. Suppose that the spacecraft weighs 1,000 kg, and that it is being launched from a railgun at the height of low Earth orbit (6.8 million meters from the center of the Earth) and traveling to the height of the Earth-Moon Lagrange point L_5 (400 million meters from the center of the Earth). Assume that $g = -9.8$ and that r is 6.4 million meters. Use the formula from part 1 (and your calculator) to compute the work needed to make this journey.

ANSWER: _____

Part 3. Suppose the spacecraft is launched at initial velocity v m/s and decelerates under the force of Earth's gravity until it stops at the L_5 point. What initial velocity is required? (You may assume that the spacecraft has no engine, and that no other forces act on the spacecraft during the journey.) Hint: Use the previous computations.

ANSWER: _____

7. (10 points) A plate of uniform density is bounded by the x-axis and the curve $y = x - x^3$, as shown below:



Find the center of mass of the plate.

(more space to work on the center of mass problem)

ANSWER: _____