MATH 2250

Midterm Exam II

November 5, 2014

NAME (please print legibly): _____

Your University ID Number: _____

Please complete all questions in the space provided. You may use the backs of the pages for extra space, or ask me for more paper if needed. This exam will be graded on:

- Correctness of computations.
- Correctness of procedure.
- Clarity of explanation of procedure.

A correct answer obtained using an incorrect or poorly explained procedure will not be graded for full credit. Please feel free to write as much as you like. Work carefully, and try to complete the problems you find easier before going back to the harder ones.

Your signature above indicates that **you understand the academic honesty policies of the University of Georgia** and **consent to photography and video recording for academic honesty purposes during the exam period**.

Good luck!

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	20	
7	10	
TOTAL	80	

1. (10 points) Find the absolute maximum and minimum values of the function

$$f(x) = 4 - x^2$$

on the interval $[-1, 3]$.

ANSWER: _____

2. (10 points) Find the indefinite integral

$$\int \csc^2 x - \csc x \cot x \, \mathrm{d}x$$

ANSWER: _____

3. (10 points) The process of "carbon dating" computes the age of a piece of organic material by the formula

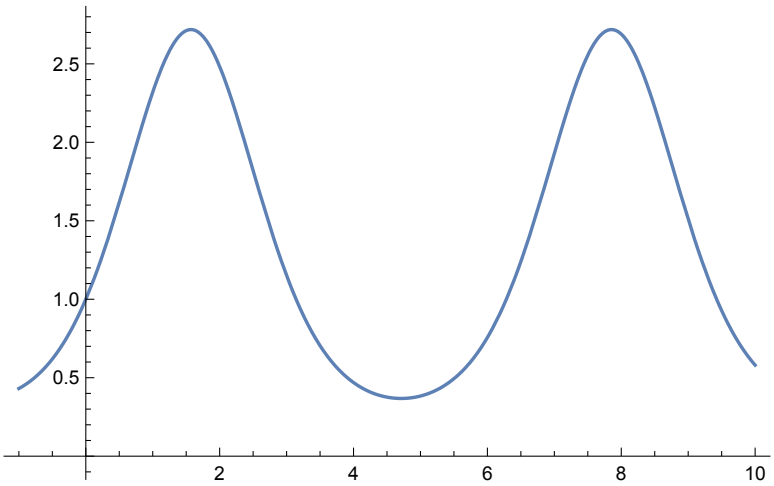
$$A(r) = 8267 \ln r \quad \text{(years)}$$

where r is the ratio of the number of atoms of a mildly radioactive isotope of carbon called "carbon-14" in an equivalent weight of living material to the number of atoms of $^{14}\mathrm{C}$ in the sample.

Suppose a chunk of wood has a measured r value of $r \sim 4 \pm 0.05$. Give the computed age of the wood sample using the formula above, and use linear approximation to estimate the error in this computation. Your answer should be in the form "x years, $\pm y$ years".

ANSWER: _____

4. (10 points)

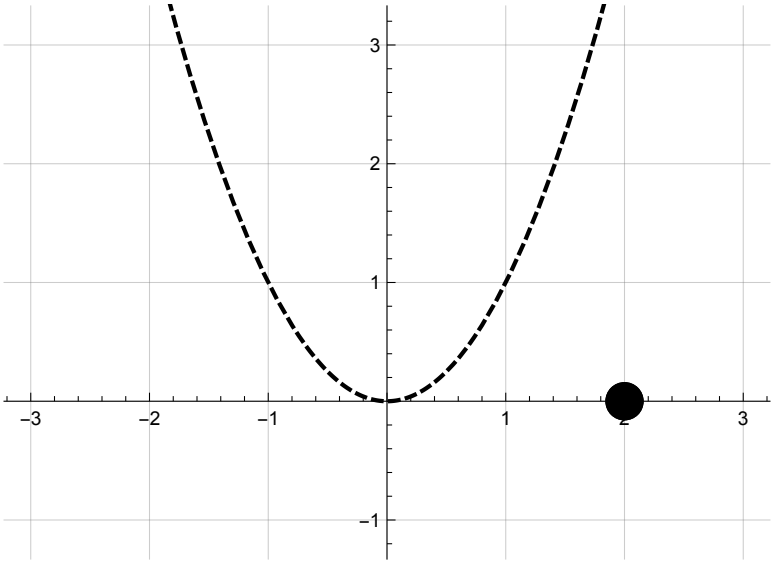


The graph above shows the *position* of the cutting head of a CNC milling machine as a function of time. On the graph, mark every time where the print head has *velocity equal to zero* with an 'x', and mark every time where the print head has *acceleration equal to zero* with an 'o'.

5. (10 points) Suppose that $f'(x) = g'(x)$. Explain in your own words why $f(x) = g(x) + C$ for some constant C. You should feel free to draw pictures and use particular examples, but you're not required to.

ANSWER: _____

6. (20 points) Find the closest point on the parabola $y = x^2$ to the marked point $(2, 0)$.



a. Write down a function $d(x)$ which gives the **square of the distance** from a point (x, x^2) on the parabola to the point $(2, 0)$.

ANSWER: _____

b. The function $d(x)$ has only one critical point c. Estimate its location to within ± 0.01 using any method you like.

(extra space to work on part b if you need it)

d. Compute the value of $d(c)$, and **go back and plot your point on the parabola on the last page.** Does this answer look reasonable to you?

ANSWER: _____

7. (10 points) At $x = 4$, we know that $f(x) > 0$, $f(x)$ has a local maximum, $f'(x)$ exists, and $f''(x)$ exists and is not zero. Suppose that $g(x) = \ln f(x)$.

Does $x = 4$ have to be a critical point for $g(x)$? Does $x = 4$ have to be a local maximum for $g(x)$? If so, why? If not, can you give an example of specific $f(x)$ and $g(x) = \ln f(x)$ functions where 4 is a local max for $f(x)$ but not a critical point or not a local max for $g(x)$?

ANSWER: _____