MATH 2250

Final Exam

December 9, 2013

NAME (please print legibly): _____

Your University ID Number: _____

A correct answer obtained using an incorrect or poorly explained procedure will not be graded for full credit. Please feel free to write as much as you like. Work carefully, and try to complete the problems you find easier before going back to the harder ones. Good luck!

Remember that you are **strongly encouraged** to have a non-graphing calculator to complete the exam. Remember also that smartphone (or computer, or other device) use is **prohibited** on this exam, regardless of what you use it for.

QUESTION	VALUE	SCORE
1	10	
2	10	
3	15	
4	20	
5	10	
6	15	
7	10	
8	10	
9	20	
10	10	
11	10	
12	10	
13	10	
TOTAL	160	

1. (10 points) Compute the limit

$$\lim_{x \to 0} \frac{x \sin x}{2 - 2\cos x}$$

using any method you like.

ANSWER: _____

2. (10 points) Please state **the definition of the derivative as a limit**:

$f'(x) =$

ANSWER: _____

3. (15 points) Compute the derivatives of the three functions below. **For the last problem, your answer should involve the unknown function $f(x)$ and its derivative $f'(x)$:**

$$\frac{d}{dx}\frac{x^3 - 2x + 4}{2x - 5} =$$

ANSWER: _____

$$\frac{d}{dx}(\arcsin 2x)(\cos x) =$$

ANSWER: _____

$$\frac{d}{dx}10^{f(x)} =$$

ANSWER: _____

4. (20 points) The ideal gas law states that the pressure $P(t)$, mass $M(t)$, and temperature $T(t)$ of a quantity of gas in a vessel of fixed volume are related by the equation $kP(t) = M(t)T(t)$ where k is a constant, t is time and T is temperature[1].

1. Differentiate this equation with respect to time t to arrive at a relationship between $P(t)$, $P'(t)$, $M(t)$, $M'(t)$, $T(t)$ and $T'(t)$.

ANSWER: _____

2. A full can of Krylon spray paint contained approximately 340 g of paint before it was used to spray "Calculus Rules" on the underside of a certain bridge. Describe the change in temperature of the paint remaining in the can during the spraying process. Justify your answer using the equation above.

ANSWER: _____

[1]I'm really sorry about using both t and T in the same problem to represent different things. Hopefully, you're used to it by now.

5. (10 points) The orbital period T of a body in an elliptical orbit with semi-major axis (maximum distance from the sun) a is given by $T = ka^{3/2}$ where k is a constant. Comet 2P/Encke has semi-major axis of 2.2178 astronomial units (AU) and an orbital period of 3.3 years.

Suppose we wish to compute the semi-major axis of 2P/Encke to an accuracy of ± 1 km by measuring its orbital period. With what accuracy ($\pm$ how many seconds?) must we measure the orbital period? Use linear approximation.

You may use the approximation 1 AU $\sim$ 150,000,000,000 m. In science, 1 year = 31,557,600 seconds (regardless of leap years and other quirks of datekeeping).

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ANSWER: _____

6. (15 points) Find the absolute maximum and minimum value of

$$y = \sqrt{4 - x^2}$$

on the interval $[-2, 1]$ using calculus. **In addition, classify any interior critical points as local maxes, local mins, or neither.**

ANSWER: _____

7. (10 points) Explain in your own words what a Riemann sum is and what it is used for. Give at least one specific example.

ANSWER: _____

8. (10 points) Consider the function

$$f(x) = x^2 \cos x$$

at $x = 2$. Is this function **increasing** or **decreasing** at this point? Is it **concave up** or **concave down**? Please use calculus to explain your answers. Grading note: Since you obviously have a 50% chance of getting the answer right by guessing, I will grade this problem **on your explanation**. A correct answer with no supporting explanation will be graded 0 points.

ANSWER: _____

9. (20 points) The graphs of the functions $y = e^{-x^2}$ and $y = x^2 - x + 1$ cross each other at the point $x = 0$ and at one other point in the interval $[0, 2]$. Use **Newton's method** or the **bisection method** (or both!) to find approximate coordinates for the second crossing point.

ANSWER: _____

10. (10 points) Find the derivative of the function

$$f(x) = \int_2^{x^2} \sin(t^3)\, dt.$$

ANSWER: _____

11. (10 points) Find the indefinite integral of the function

$$f(x) = x^3 - 4x + 3.$$

ANSWER: _____

12. (10 points) Evaluate the integral

$$\int_4^{100} \frac{1}{2\sqrt{x}}\, dx.$$

ANSWER: _____

13. (10 points) Evaluate the indefinite integral

$$\int e^{\sin x} \cos x \, dx.$$

ANSWER: _____