MATH 2250

Midterm Exam II

April 5, 2012

NAME (please print legibly): _____

Your University ID Number: _____

Please complete all questions in the space provided. You may use the backs of the pages for extra space, or ask me for more paper if needed. This exam will be graded on:

- Correctness of computations.
- Clarity of explanation of procedure.
- Correctness of procedure.

A correct answer obtained using an incorrect or poorly explained procedure will not be graded for full credit. Please feel free to write as much as you like. Work carefully, and try to complete the problems you find easier before going back to the harder ones. Good luck!

Remember that you are **required** to have a non-graphing calculator to complete the exam. Remember also that smartphone (or computer, or other device) use is **prohibited** on this exam, regardless of what you use it for.

QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	15	
TOTAL	75	

1. (10 points) The *linear* approximation to $f(x + \Delta x)$ (a linear polynomial in Δx) is given by

$f(x + \Delta x) \simeq$

The *quadratic* approximation to $f(x + \Delta x)$ (a quadratic polynomial in Δx) is given by

$f(x + \Delta x) \simeq$

Taylor's theorem says that $f(x + \Delta x)$ has a polynomial approximation of any degree n. Write down the general form of Taylor's theorem (a polynomial of degree n in Δx)

$f(x + \Delta x) \simeq$

2. (10 points) Suppose we know that $f(7) = 4$, $f'(7) = 6$, $f''(7) = 2$ and $f'''(7) = 3$. Give the BEST estimate you can for $f(7.2)$ using this information and Taylor's theorem.

ANSWER: _____

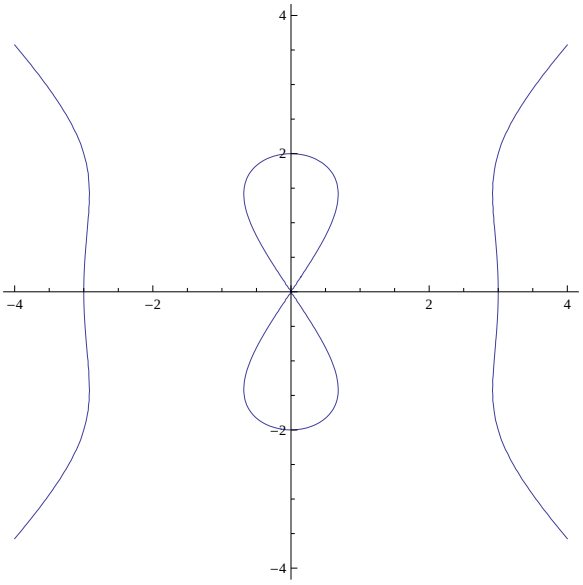
3. (10 points) The function

$$g(x) = \frac{x^3}{3} - 3x^2 + 8x$$

has two critical points, each of which is either a local max or a local min. Where are they? Use the second derivative test to tell whether they are local maxes or local mins.

ANSWER: _____

4. (10 points) Find the equation for the tangent line to the curve described by



$$y^4 - 4y^2 = x^4 - 9x^2$$

at the point $(3, 2)$ using implicit differentiation (point-slope form is fine– you don't need to convert to slope-intercept form. You also don't have to simplify your expression for slope.).

ANSWER: _____

5. (10 points) A photographic flash for a high speed camera produces

$$f(x) = 150 \cdot 10^x$$

lumens of light at 5 feet with an input power of x kilovolts. The film requires an illumination of $150 \cdot 10^2 \pm 10^2$ lumens at 5 feet to get the correct exposure (too much light will overexpose the picture and too little will underexpose the picture).

Use linear approximation and Taylor's theorem to find V and ΔV so that a power supply which produces $V \pm \Delta V$ kilovolts of output should result in an acceptable output of light from the flash. **HINT:** Remember that $\frac{d}{dx}10^x$ is **NOT** 10^x.

ANSWER: _____

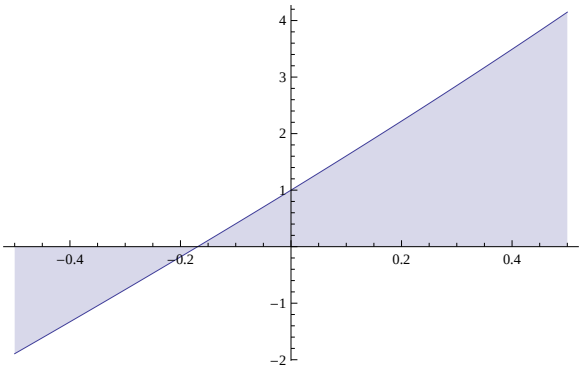
6. (10 points) Find the maximum and minimum of the function

$$f(x) = e^{2x} + e^{-x}$$

on the interval $[-1, 0]$. Please express your answer as a decimal approximation, rounded to two digits after the decimal point (e.g. 47.2233485 would be rounded to 47.22). You will need to use your calculator, but this is *not* a Newton's method problem. **Algebra Hint:** $e^{2x} = e^{3x-x} = e^{3x}e^{-x}$.

ANSWER: _____

7. (15 points) Consider the function $f(x) = e^x + 5x$, whose graph is



Solve for the solution of $f(x) = 0$ shown in the graph using your calculator and Newton's method. Your score on this problem will be 3 times the number of correct digits (to the right of the decimal point) in your answer, with a maximum of 15 points.

Hint: I'm testing arithmetic here as well as calculus. So please be careful about your work! Keep in mind that you can always **check** your answer by plugging it back into the original function.

Please continue your work for Problem 7 on this page if needed.

ANSWER: _____